An Eigenvector-Supported Optimization Method for Holographic-Based Leaky Wave Antennas

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Abstract—A novel eigenvector-supported optimization method for holographic-based leaky wave antennas is presented. To assign the analytical impedance tensor hologram onto a pixel geometry, a 3D-eigenmode tensor database is utilized. The optimization process calculates the pixel orientation angles of the analytical tensor and the assigned eigenmode impedance tensor based on their respective eigenvectors for each unit cell. If a pixel angle deviation is observed, the correction algorithm is applied to minimize the error between the analytical tensor and the eigenmode tensor. This results in an optimized impedance tensor hologram and a more accurate realization of the anisotropy degree through the corresponding pixel geometry parameters. Employing the eigenvector-based optimization approach, a holographic-based leaky wave antenna is fabricated on a fused silica wafer for an operational frequency of 160 GHz. The measured far field pattern shows a gain of 33.2 dBi, leading to an aperture efficiency of 34.6 % and a side-lobe level of −29 dB at 160 GHz.

Index Terms—holographic antenna, impedance tensor, metasurface antenna, leaky wave, surface wave, high-gain, glass technology.

I. INTRODUCTION

Holographic-based leaky wave antennas (HLWAs) are gaining an increasing interest in the mm-wave community, since they enable a flexible control of the electric aperture field distribution regarding amplitude, phase, and polarization of the radiating electromagnetic wave in the far field. They are based on the holographic principle allowing the transformation of a bounded surface wave (SW) mode into a continuously radiating leaky wave (LW) mode. This can be achieved through the interaction of the SW with a dense periodic structure of sub-wavelength discontinuities, which is described by the space-dependent impenetrable impedance boundary condition (IIBC) [1], [2]. The periodic structure is placed on a grounded dielectric slab, since the modulated impedance surface (MIS) hologram is TM-based and the propagation property of the fundamental TE0 SW mode is suppressed [3]. In previously published works, the assignment of the analytical impedance tensor to a physical pixel geometry is implemented by a scalar MIS synthesis method [4], [5]. Unfortunately, the downside of the scalar hologram synthesis results in a poor polarization purity. A novel method to extract the surface impedance tensor of an arbitrary shaped pixel geometry by solving a non-linear system of equations (NLSoE) is presented in [6]. Subsequently, a 3D-eigenmode tensor database is created for an impedance tensor hologram synthesis [6]. This enables an even more accurate realization of the anisotropy degree to a corresponding pixel geometry compared to the scalar synthesis approach.

In this contribution, a novel eigenvector-based optimization method for HLWAs is presented and leads to a further improvement of the anisotropy degree of the tensor hologram resulting in an enhanced antenna gain, polarization purity, and side-lobe level. For an evaluation of the optimization process of the tensor hologram synthesis, the dispersion curve error is introduced. A HLWA prototype is realized on a fused silica wafer due to its low losses at mm-wave frequencies beyond 140 GHz [7].

II. SYNTHESIS FOR HOLOGRAPHIC-BASED LEAKY WAVE ANTENNAS

The holographic principle describes the generation of an MIS hologram as an interference pattern of a bounded SW propagating on a interface between two dissimilar materials and continuously radiating LW mode. The synthesis is based on the interaction of the SW with the MIS hologram, which is realized by a dense periodic structure of sub-wavelength discontinuities. This is mathematically described by the IIBC. For this purpose, a surface wave launcher (SWL) excites the fundamental TM0 SW mode propagating rotationally symmetric on the antenna aperture [4]. Its transverse magnetic field distribution \( \vec{H}_{\text{TM0}}\mid _{z=0^+} \) follows the Hankel function of the second kind of first order [2]. The IIBC links the transverse magnetic field \( \vec{H}_{\text{TM0}}\mid _{z=0^+} \) to the transverse electric field distribution \( \vec{E}_{\text{TM0}}\mid _{z=0^+} \) on the antenna aperture through the surface impedance tensor \( \vec{Z} \) [8]

\[
\vec{E}_{\text{TM0}}\mid _{z=0^+} = \vec{Z} \cdot \left( \vec{e}_z \times \vec{H}_{\text{TM0}}\mid _{z=0^+} \right),
\]

where \( \vec{e}_z \) describes the unit vector normal to the antenna aperture. The objective aperture field distribution has the form

\[
\vec{E}_{\text{obj}}(x, y) = \vec{T}(x, y) \circ \vec{E}_{\text{0}} \circ \left( e^{-j(k_x x)} + e^{-j(k_{\gamma} y + \frac{\pi}{2})} \right),
\]

with \( \circ \) being the Hadamard product and \( \vec{T}(x, y) \) defines the \( x \)- and \( y \)-components of the desired planar amplitude taper function. An arbitrary polarization is provided by the complex phase term, but in this work a circularly polarized excited electric field is used. The constant space independent field amplitudes are described by \( \vec{E}_{\text{0}} \). The spatial decay of the impedance tensor hologram \( \vec{Z}(x, y) \) in the \( x-y \)-plane
is described by the modulation indices $M_x$ and $M_y$ [9]. The synthesized electric field distribution resulting from (1) comprises all reactive field components including those of the radiating $(-1,-1)$-LW mode whose spectrum is completely inside the visible region [10], [11].

III. TENSOR HOLOGRAM SYNTHESIS

The MIS hologram is realized by a dense periodic structure of sub-wavelength discontinuities. This is implemented by a discrete set of unit cells (UCs), each with a pixel placed in its center. The pixel geometry is based on a slot-loaded elliptically shaped patch, as can be seen in [5]. The pixel geometry parameters are defined by the pixel width $w_{pxl}$ and the pixel orientation angle $\phi_{pxl}$.

A. Impedance Tensor Hologram Synthesis

In order to realize the analytical tensor hologram resulting from Sec. II by a discrete set of UCs, the impedance tensor hologram synthesis presented in [6] is performed. Thus, an eigenmode tensor look-up table (LUT) $\mathbb{Z}_{\text{ideal}}$ must be created by solving the angle-dependent NLSoE for $M$ orientation angles and $N$ pixel widths $(\phi_{pxl,m}, w_{pxl,n})$ [6]. The goal is to find the minimum impedance tensor error $\epsilon^r$ between the ideal analytical tensor $\mathbb{Z}_{\text{ideal}}$ and the eigenmode tensor LUT to determine the physical pixel geometry parameters for every UC

$$\epsilon^r(\phi_{pxl,m}, w_{pxl,n}) = \left| \mathbb{Z}_{\text{ideal}}(\phi_{pxl,m}, w_{pxl,n}) - \mathbb{Z}_{\text{ideal}} \right|.$$  \hspace{1cm} (3)

Subsequently, the pixel geometry results from the impedance tensor error having the lowest error entries across all four tensor components

$$\epsilon^r(\phi_{pxl,m}, w_{pxl,n}) = \arg\min_{\phi_{pxl,m}, w_{pxl,n}} \left\{ \epsilon^r(\phi_{pxl,m}, w_{pxl,n}) \right\}. \hspace{1cm} (4)$$

Hence, the corresponding eigenmode impedance tensor $\mathbb{Z}_{\text{eig}}$ can be determined from the LUT by using the already determined pixel geometry parameters

$$\mathbb{Z}_{\text{eig}} = \mathbb{Z}_{\text{eig,LUT}}(\phi_{pxl,\text{eig}}, w_{pxl,\text{eig}}). \hspace{1cm} (5)$$

This synthesis method is performed for every UC to realize the analytical impedance tensor hologram, which synthesizes the electric field distribution by using the IIBC (1).

B. Eigenvector-Based Optimization Approach

The tensor hologram synthesis from Sec. III-A primarily depends on an accurate solution of the NLSoE for each pixel geometry. This ensures that the calculated impedance tensor entries and their corresponding pixel geometry parameters in the LUT are correctly assigned to each other. In order to accurately verify the assignment of the analytical tensor $\mathbb{Z}_{\text{ideal}}$ to the eigenmode tensor $\mathbb{Z}_{\text{eig}}$ resulting from Sec. III-A, an eigenvector-based (EV-based) optimization process is performed for every UC (see Fig. 1).

The pixel orientation angles of the ideal analytical tensor $\phi_{pxl,\text{ideal}}$ (see \(\text{①}\) in Fig. 1) and the assigned eigenmode tensor $\phi_{pxl,\text{eig}}$ (see \(\text{②}\) in Fig. 1) are computed by the eigenvector approach from [5] (see \(\text{③}\) in Fig. 1). This results in an ideal pixel orientation angle $\phi_{pxl,\text{ideal}}$ and in the pixel orientation of the eigenmode tensor $\phi_{pxl,\text{eig}}$. Subsequently, the L1 Norm $\|\cdot\|_1$ is used to calculate the angle deviation $\phi_{rot}$ between the EVs of the analytical and the eigenmode tensor

$$\phi_{rot} = \left\| \left( \phi_{pxl,\text{ideal}}, \phi_{pxl,\text{eig}} \right) \right\|_1. \hspace{1cm} (6)$$

This equals the rotation angle $\phi_{rot}$ of their principal axis in the $\mathbb{R}\{k_x\} - \mathbb{R}\{k_y\}$-plane as depicted in Fig. 2. In case the criterion (see \(\text{④}\) in Fig. 1)

$$\phi_{rot} \pmod{180^\circ} \equiv 0^\circ \hspace{1cm} (7)$$

is fulfilled, the assigned pixel geometry parameters and their corresponding eigenmode tensor resulting from tensor hologram synthesis method (Sec. III-A) are maintained, and no correction is necessary.

Otherwise, the correction algorithm is performed (see \(\text{⑤}\) in Fig. 1). In this process, the primary assigned pixel orientation $\phi_{pxl,\text{eig}}$ is shifted by $\phi_{rot}$ and results in $\phi_{pxl,\text{opt}}$ as depicted in Fig. 3. This corrects the pixel angle deviation of the dispersion curves between the ideal tensor (① in Fig. 1) and the eigenmode impedance tensor.

![Fig. 1: Block diagram of the eigenvector-based optimization method for the pixel orientation angle.](image-url)
(2) in Fig. 1). Subsequently, a $1 \times N \times 4$ tensor LUT results containing all four tensor components $Z_{xx}, Z_{xy}, Z_{yx}$ and $Z_{yy}$ at the one optimized pixel orientation angle $\phi_{\text{opt}}$ for $N$ different pixel widths

$$\langle \phi_{\text{opt},x}, u_{\text{opt},n} \rangle = \left| \sum_{l=0}^{1} \gamma_{\text{opt},l} \langle \phi_{\text{opt},x}, u_{\text{opt},n} \rangle - \gamma \right| \ .$$

(8)

The minimum impedance tensor error among the $N$ different tensor entries gives the corresponding optimized pixel width $u_{\text{opt}}$ (see Fig. 3)

$$\langle \psi_{\text{opt},x}, u_{\text{opt},n} \rangle = \arg \min_{u_{\text{opt},n}} \left\{ \psi_{\text{opt},x} \langle \psi_{\text{opt},x}, u_{\text{opt},n} \rangle \right\} \ .$$

(9)

Afterwards, the optimized pixel geometry and the associated eigenmode impedance tensor $\gamma_{\text{opt},x}$ can be determined from the tensor LUT by using the optimized pixel parameters $\langle \phi_{\text{opt},x}, u_{\text{opt},n} \rangle$ (see 6 in Fig. 1)

$$\gamma_{\text{opt},x} = \sum_{l=0}^{1} \gamma_{\text{opt},l} \langle \phi_{\text{opt},x}, u_{\text{opt},n} \rangle \ .$$

(10)

This novel EV-based optimization process is performed for every UC on the antenna aperture to realize the analytical impedance hologram.

### C. Dispersion Curve Error

In order to evaluate the improvement of the EV-based optimization method regarding the tensor hologram synthesis, the dispersion curve of the impedance tensor is considered.

The dispersion curve error for each $l$-th incident angle $\phi_{\text{inc},l}$ w.r.t. to the UC is defined as

$$\epsilon_{\text{D},l} = J Z_{\text{F0}} \left( \sqrt{\left( \frac{\beta_{\text{ideal},l}}{K_0} \right)^2 - 1} - \sqrt{\left( \frac{\beta_{\text{eig},l}}{K_0} \right)^2 - 1} \right) \ ,$$

(11)

with $\beta_l = \sqrt{\Re{\{k_x (\phi_{\text{inc},l})\}}^2 + \Re{\{k_y (\phi_{\text{inc},l})\}}^2}$ and $Z_{F0}$ being the impedance of free space. A vivid illustration of the misalignment between the ideal tensor (→) and the eigenmode tensor (→) is depicted in Fig. 2, where $\epsilon_{\text{D},l}$ is highlighted by the gray area (≡). The dispersion curve root mean square (RMS) error $\epsilon_{\text{D},\text{RMS},\text{UC}}$ across the whole incident angle range $L$ from $0^\circ$ to $360^\circ$ for each UC results to

$$\epsilon_{\text{D},\text{RMS},\text{UC}} = \sqrt{\frac{1}{L} \sum_{l=0}^{L} |\epsilon_{\text{D},l}|^2} \ \forall l \in [0, 360] \ .$$

(12)

The global RMS error of the whole impedance tensor hologram $\epsilon_{\text{D},\text{RMS},\text{Holo}}$ consisting of $I$ UCs is calculated as

$$\epsilon_{\text{D},\text{RMS},\text{Holo}} = \sqrt{\frac{1}{IL} \sum_{l=0}^{I} \sum_{l=0}^{L} |\epsilon_{\text{D},l}|^2} \ \forall l \in [0, 360] \land \forall i \in \mathbb{N} \ .$$

(13)

### IV. HLWA Realization and Measurement Results

#### A. Design Process and Analytical Comparison

Based on the EV-optimized tensor hologram synthesis discussed in Sec. III-B, a HLWA is designed. In order to suppress higher order SW modes, the thickness of the glass wafer is set to $160 \mu m$. A coplanar waveguide (CPW) within
the ground metallization is used to feed the SWL from the bottom side. The dimensions of the CPW correspond to a characteristic impedance of 50 Ω. For a meaningful comparison between the tensor hologram synthesis method from Sec. III-A and the EV-optimized tensor hologram synthesis approach from Sec. III-B a HLWA operating at 160 GHz is designed for both synthesis methods. The antenna is intended to radiate a right-hand circularly polarized (RHCP) pencil beam in boreside direction under \((\phi_0 = 0°, \phi_0 = 0°)\). A 2D-Taylor window is used for the planar amplitude taper function \(T(x, y)\) of the desired objective aperture field (2) for a theoretical side-lobe level (SLL) \(\geq 32\) dB in the analytical far field pattern. The SWL is placed in the center of the circularly shaped antenna aperture at \((x_0 = 0\, \text{mm}, y_0 = 0\, \text{mm})\). The MIS hologram consists of 44,000 squared UCs, and to ensure a homogeneous periodic structure, the UC size is set to \(0.2\, \text{mm} \times 0.2\, \text{mm}\) resulting in an aperture area of \(1800\, \text{mm}^2\). The proposed UC configuration provides an average impedance of \(Z_{\text{avg}} = j256\, \Omega\) and a maximum modulation index of \(\max\{\tilde{M}_{x,y}\} = 0.28\).

In Fig. 4 the RMS dispersion error vs. the MIS consisting number of UCs calculated with (11) - (12) is shown. The EV-based optimization of the tensor hologram synthesis method leads to a significant reduction of the RMS dispersion curve error \(e_{\text{D,RMS,UC}}\) of at least 3.5Ω corresponding to a significant error reduction of more than 20% per UC (see Fig. 4). The global dispersion curve RMS error \(e_{\text{D,RMS,Holo}}\) (13) of the full impedance tensor hologram decreases by 90Ω due to the EV-based optimization of the pixel orientation angle. Thus, a lower pixel angle deviation \(\phi_{\text{rot}}\) (6) exists, leading to a more accurate implementation of the analytical impedance tensor hologram. The synthesized eigenmode tensor components \(\sum_{i} e_{\text{ig,opt}}\) which results from the EV-optimized tensor hologram synthesis (Sec. III-B) are depicted in Fig. 6. The co-polarization and cross-polarization of the IIBC simulated far field patterns are depicted in Figs. 5a and 5b. The radiation pattern based on the EV-optimized tensor synthesis approach shows an enhancement of the polarization purity of up to 7 dB due to the optimization of the pixel orientation angle, leading to a more accurate implementation of the anisotropic degree. This also reflects a more precise synthesis of the desired objective electric aperture field distribution resulting from the IIBC (1). Thus, an SLL of at least 32 dB and an analytical antenna gain of 33.5 dBi, which corresponds to an enhancement of 1.2 dB compared to the tensor hologram synthesis from Sec. III-A at 160 GHz are achieved by the EV-based optimization method.

**B. Realization and Measurements**

The EV-optimized HLWA on glass was fabricated on a fused silica wafer with a dielectric loss tangent of \(\tan\delta = 0.001\) and a relative permittivity of \(\varepsilon_r = 3.78\). The metallization for dense periodic pixel structure consists of a 10 nm thick chromium seed layer as an adhesive layer, on which a 350 nm gold layer is added. The individual pixel structures were realized by a conversational wet etching process with maximum tolerances up to 2 μm. A photo of the HLWA prototype is given in Fig. 7. The proposed HLWA prototype is excited from the bottom side by a probe tip in upside-down orientation to avoid the parasitic impact of the probe on the radiation pattern of the antenna under test (AUT). For the seek of conciseness, the measurement setup is exactly the same as in [4]. The far field measurements of the realized holographic antenna are conducted using a robot-supported mm-wave antenna measurement setup. The measured radiation patterns of the realized HLWA are illustrated in Fig. 8 for elevation angles up to \(\pm 30°\) at the operation frequency of 160 GHz at the azimuth angles 0° and 90°. The EV-based optimization method leads to an improvement of the measured peak antenna gain from 32 dBi to 33.2 dBi. This results in an aperture efficiency (AE) of 34.6%. A polarization purity of 45 dB can be determined from the radiation patterns, which corresponds to a significant improvement of 25 dB compared to the scalar method. A SLL of 29 dB were measured, whereby a reduced SLL decay is present due to radiation at the wafer edges. The input reflection
In this paper, a novel eigenvector-based optimization method for HLWAs has been presented. First, a 3D-impedance tensor database is created in order to assign the analytical method for HLWAs. Then, an anisotropic HLWA is realized on a fused silica wafer. The measured radiation patterns show an antenna gain of 33.2 dBi, a side-lobe level of at least 29 dB, and a polarization purity of 45 dB at an operation frequency of 160 GHz. This corresponds to an AE of 34.6%.

**REFERENCES**


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![Fig. 7: Photograph of the realized HLWA prototype on glass.](image)

![Fig. 8: Measured radiation patterns of the anisotropic HLWA on glass at 160 GHz: (a) co-pol ($\phi_0 = 0^\circ$) and cross-pol ($\phi_0 = 90^\circ$) and (b) co-pol ($\phi_0 = 0^\circ$) and cross-pol ($\phi_0 = 90^\circ$).](image)

algorithm is performed, resulting in an optimized eigenmode tensor and its optimized pixel geometry parameters. Based on the eigenvector-supported impedance tensor synthesis method, an anisotropic HLWA is realized on a fused silica wafer. The measured radiation patterns show an antenna gain of 33.2 dBi, a side-lobe level of at least 29 dB, and a polarization purity of 45 dB at an operation frequency of 160 GHz. This corresponds to an AE of 34.6%.

V. CONCLUSION

In this paper, a novel eigenvector-based optimization method for HLWAs has been presented. First, a 3D-impedance tensor database is created in order to assign the analytical impedance tensor to a physical pixel geometry and its corresponding eigenmode impedance tensor. The eigenvectors of the impedance tensors were used to determine their pixel angle deviation. Subsequently, a pixel angle correction