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Investigation of the Effects of Noise on a Frequency Synthesizer

Gurpreet Singh Sangha, Michael H.W. Hoffmann

Institute of Microwave Techniques, University of Ulm, Germany

Abstract: This paper contains a rigorous noise analysis of a second order PLL based synthesizer circuit. In contrast to previously done analysis, a more complete combination of noise sources is considered. A new model for frequency divider and an alternative non-standard noise model are chosen. This modeling technique results in the derivation of a very general mathematical model for the output of synthesizer taking into account noise sources at different points in the circuit. This mathematical description shows clearly how the noise at different points in the circuit affects the overall behavior of the synthesizer.

Keywords: Synthesizer, PLL, Noise, Modeling, Simulations, Matlab, Simulink
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INTRODUCTION

For a good design of a PLL based frequency synthesizer, the investigation of effect of electronic noise is an important factor to be considered and analyzed. Electronic noise can result in random modulation of the phase of an oscillator's output due to a variety of internal and external noise sources.

This paper contains a detailed noise analysis of a PLL based synthesizer circuit considering effects of internal and external noise sources. Theoretical analysis methodology and results from nonlinear simulations will be presented.

Many valuable works have been done on the noise behavior of a PLL using different models which included some effects of noise [1]-[5]. This paper contributes to this field by doing a rigorous synthesizer circuit noise analysis by taking into account a more complete combination of noise sources. This work is a continuation of previously published work by the same authors [7],[8].

NOISE ANALYSIS METHODOLOGY

Figure 1 shows a phase-model of a second order PLL that was used for this work. As expected, it consists of a phase-detector (PD), a control filter (CF), and a voltage controlled oscillator (VCO). The control filter used is a first order lag-lead type.

In contrary to the commonly used model for the frequency divider consisting of hard-limiter and bandpass filter, a new superior model is used as described in Figure 2. The new piecewise linear limiter model represents a real limiter of a frequency divider in a better way.

Given a sinusoidal signal $R(t)\cos(\omega_c t+\varphi_c+n_i(t))$ at the input, the output signal of this divider can be derived by application of Fourier-series method with respect to the phase. The divider output bandpass signal can be written as show in equation (1).

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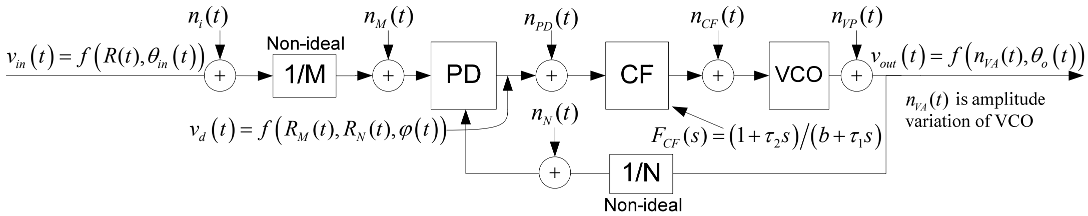

FIGURE 1. Synthesizer model used for the noise analysis.

$$u_M(t) = \left\{ \frac{4b}{\pi} \sin(\theta) - \frac{b}{a\pi} \sin(2\theta) R(t) \right\} \cos\left(\frac{\omega_c}{M} t + \frac{\varphi_c}{M} + \frac{n_i(t)}{M} \right) = R_M(t) \cos(\theta_M(t)) \tag{1}$$

with the conduction angle θ=arccos (a/R_L), and R_L being the limiting amplitude value. Parameters a and b describe the *x-y-axis* amplitude limiting factors.

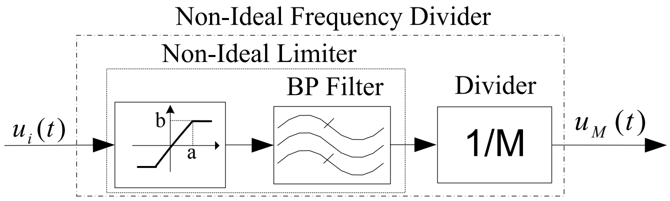

FIGURE 2. Non-ideal frequency divider where a nonlinear limiter is approached by using piecewise linear characteristic.

In contrast to the ideal limiter which suppresses completely the amplitude fluctuations of the input signal, the output signal in this case is dependent on the amplitude variations of the input signal as it would be expected from a real divider. This model is more general than the one used in the literature [9]. For $a=0$, which is the case of an ideal limiter, this model gives results that are in agreement with the ideal limiter model, i.e. $R_M(t)=4b/\pi$.

Using the results from above, a model for the output of the phase detector can be formed where output voltage v_d is described as a function depending not only on the phase-difference φ between VCO-signal phase fluctuations θ_o and input-signal phase fluctuations θ_i but also on amplitude fluctuations $R_M(t)$ of input signal, as well as of amplitude fluctuations $R_N(t)$ of the VCO signal. The novel model shown below is thus more complete as compared to models in use:

$$v_d(t) = g(R_M(t), R_N(t), \varphi(t)) \tag{2}$$

A very complex combination of noise sources is considered here:

1. At the input, an additional noise source of the divider M is considered together with amplitude and phase noise $R(t)$ and $n_i(t)$ of the input signal. Input amplitude fluctuations $R(t)$ result in a proportional term $R_M(t)$ in the phase detector function v_d after the divider M. The noise source at the input is taken to be white that is converted into amplitude and phase components.
2. The two noise sources $n_{PD}(t)$, $n_{CF}(t)$ represent the noise of the PD and the CF, respectively. These sources are taken to be a superposition of $1/f$ and white noise.
3. The VCO noise can be divided into amplitude and phase components $n_{VA}(t)$ and $n_{VP}(t)$, respectively. VCO amplitude fluctuations $n_{VA}(t)$ result in a proportional

term $R_N(t)$ in the phase detector function v_d after the divider N. Noise of the VCO is also taken to be a superposition of $1/f$ and white noise.

4. At the output of the divider N there is an additional noise source $n_N(t)$.

In order to describe the noise at each point mathematically, a non-conventional noise model described by the following random process is used:

$$u(t) = A_0/2 + \sum_{n=1}^{M} A_n \cos\left(n\Delta\omega t + \varphi_n\right) \tag{3}$$

with $\Delta\omega=2\pi/T_{ob}$; T_{ob} is the observation time. This non-standard noise model can be traced back to S.O.Rice [6]. The important difference in this model is that the terms A_n are deterministic and the terms φ_n are random variables which are jointly uniformly distributed in the interval $[0,2\pi)$. This less frequently used model is mathematically simpler than the standard model and by virtue of the central limit theorem it can be shown that it leads to the same results as the more commonly used model. The terms A_n from equation (3) allow for designing the noise to have different types of spectra.

The nonlinear differential equation of this synthesizer system is described similar to [7]. The analysis of such a nonlinear differential equation is usually mathematically very complex. The authors have already presented a novel linearization approach in [7]. Using this approach, expressions for the synthesizer output voltage and output average power are derived. These are rather complex formulas and will not be presented in this paper.

SIMULATION RESULTS

In addition to the mathematical formulations, simulation using SIMULINK was performed. Only the most important results from the nonlinear model simulations are shown here.

Diagrams (a), (b), (c) and (d) in Figure 3 shows the log-log output power spectrum of the synthesizer circuit with noise at different points. As it was expected from theoretical results (not shown here) simulations show clearly how the noise at different points in the circuit affects the output of the synthesizer. The two dividers help to decrease the noise of the input and the VCO signal, respectively. A white noise at the input causes a flatting of the output spectrum around the carrier as shown in (a). PLL helps to suppress the VCO noise within the loop bandwidth as indicated in (b). The noise source in front of the VCO plays a critical role and can lead to strong phase noise modulations in the output. The amplitude fluctuations as expected do play a significant role in the output spectra forming. The asymmetry of the amplitude fluctuation spectrum indicates that amplitude variations results in mixing of amplitude and phase noise to the output signal.

The angular frequency of the input signal is chosen to be $50{\cdot}10^3$ rad/sec and the dividers are chosen as M=2 and N=4.

CONCLUSION

This paper contains a more complete noise analysis of a PLL based synthesizer circuit considering complex combination of noise sources. In addition to the use of the non-standard Rice's noise model, a new and improved model for the frequency divider is used that helps to model the effects of amplitude fluctuations at the output. This new setup leads to a deeper understanding of the complex noise phenomenon affecting the output of the synthesizer. It shows how each point in the synthesizer circuit is contributing to the overall noise behavior.

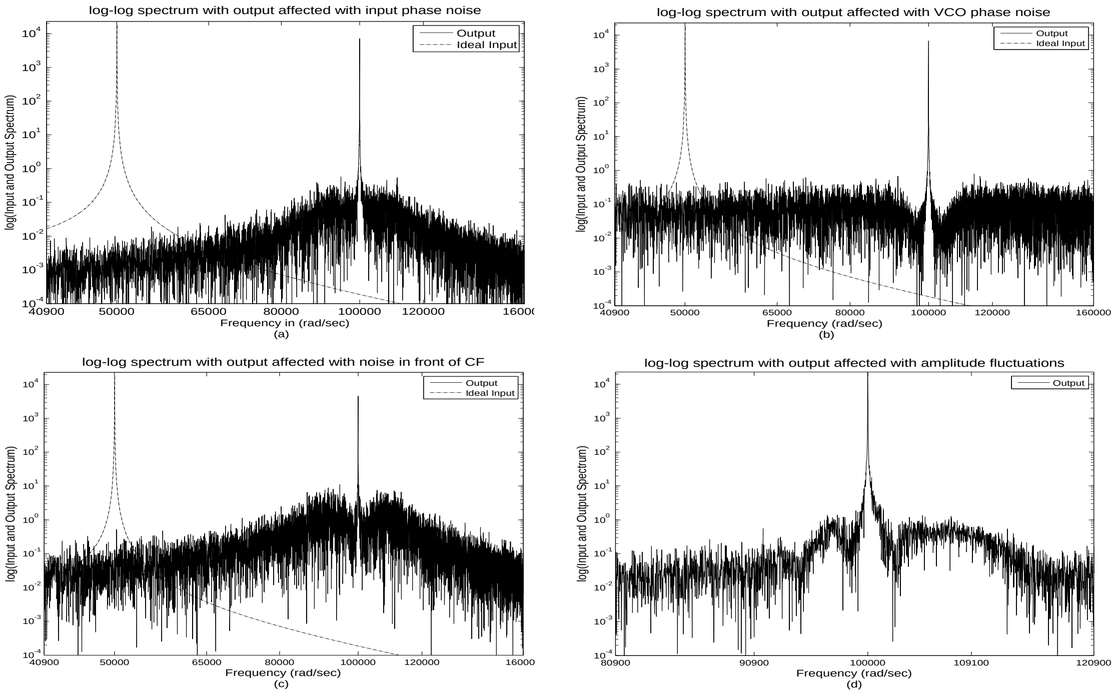

FIGURE 3. Graphs (a), (b), (c) and (d) show the power spectrum of the input and the output with noise present at different points of the synthesizer.

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