

On the Relation between Composite Right/Left-Handed Transmission Lines and Chebyshev Filters

Changjun Liu and Wolfgang Menzel

Abstract— Composite right/left handed (CRLH) transmission lines have gained great interest in the microwave community. In practical applications, such CRLH sections realized by series and shunt resonators have a finite length. Starting from the observation that a high order Chebyshev filter also exhibits a periodic central section of very similar structure, the relations between finite length CRHL transmission lines and Chebyshev filters are discussed in this paper. It is shown that a finite length CRLH transmission line in the balanced case is equivalent to the central part of a low-ripple high-order Chebyshev band-pass filter, and a dual-CRLH transmission line in the balanced case is equivalent to a low-ripple high-order Chebyshev band-stop filter. The nonperiodic end sections of a Chebyshev filter can be regarded as matching sections, thus leading to an even better amplitude and phase response. It is also shown that, equally to a CRHL transmission line, a Chebyshev filter exhibits negative phase velocity in part of its passband. As a consequence, an improved behavior of finite length CRLH transmission lines may be achieved adding matching sections based on filter theory; this is demonstrated by a simulation example.

Index Terms—Band-pass filters, band-stop filters, Chebyshev filters, metamaterials, negative refractive index

I. INTRODUCTION

IN the past years, metamaterials [1] have found wide interest in the microwave community. A homogeneous negative refractive index, left-handed, or metamaterial transmission line, however, does not exist in nature and typically is approached by an artificial structure which is usually constructed from a series of discontinuous sections and operates in a restricted frequency range [2,3]. When the dimension of such sections is much smaller than the wavelength at the operating frequencies, such transmission lines are considered as homogeneous media. A typical realization is found in a quasi-lumped transmission line with elementary cells consisting of series capacitors and shunt inductors [4]. As the normal shunt capacitance and series inductance cannot be avoided in practice, the concept of a composite right/left-handed (CRLH) transmission line was developed, and a number of applications have been demonstrated [4-9], e.g. for frequency scanned antennas, directional couplers, power dividers, zeroth order resonators etc., and further applications of negative refractive index

transmission lines for constructing advanced microwave components [10,11] are envisaged.

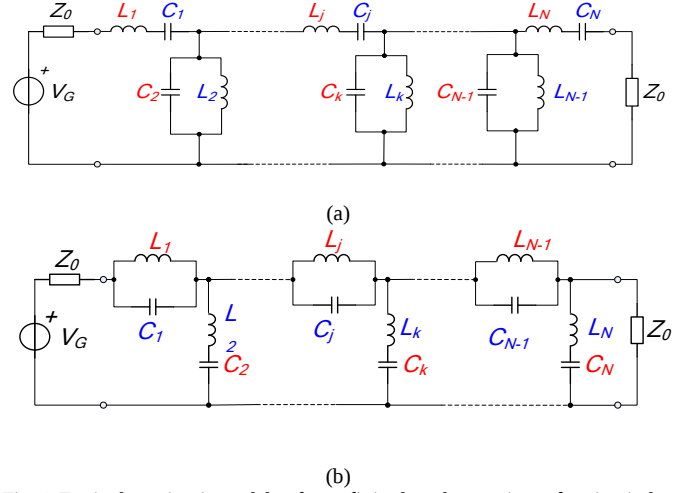


Fig. 1 Equivalent circuit models of two finite length negative refractive index transmission lines. (a) CRLH transmission line. (b) Dual-CRLH transmission line.

Fig. 1(a) shows the equivalent circuit of a (finite length) CRLH transmission line (connected to a source generator on the left side and a load at the right side). Looking at this equivalent circuit, this CRLH transmission line resembles a band-pass filter of typically high order. In the following, we limit our discussions to such one-dimensional CRLH transmission lines.

In a dual-CRLH transmission line, the series-branch and the shunt-branch resonators of a CRLH transmission line are interchanged resulting in a different type of negative refractive index transmission line [13]. Fig. 1(b) shows the circuit model of such a dual-CRLH transmission line, which, in this case, looks like a high-order band-stop filter.

A conventional filter, on the other hand, is generally not a uniform structure. However, looking at the filter coefficients g_i of a Chebyshev filter, such a filter with high order exhibits a highly periodic central section. From a different point of view, it also can be considered as a periodic structure with impedance matching sections at both ends (see Fig. 3). In [12] we have briefly shown that the CRLH transmission line may be equivalent to a part of the Chebyshev band-pass filter, which was done partly in an intuitive way.

The starting points of our considerations were on the one hand, the periodic nature of the central part of Chebyshev filters, and on the other hand, practical problems of implementing finite length CRLH transmission lines into a (typically) 50 Ohm environment. Therefore, we wanted to find

C. Liu is with the School of Electronics and Information Engineering, Sichuan University, Chengdu 610064, China. (e-mail: cjliu@ieee.org).

W. Menzel is with the Institute of Microwave Techniques, University of Ulm, Ulm D-89069, Germany. (e-mail: wolfgang.menzel@uni-ulm.de).

the relations between CRLH transmission lines and Chebyshev filters to design better CRLH transmission line sections with the help of classical filter theory.

Typically, CRLH transmission lines are considered as tools for dispersion engineering – this is in some way equivalent to frequency dependent phase control. Amplitude plays a minor role, although the practical implementation of a finite length section of a CRLH transmission line (and only this is feasible in reality) leads to problems with impedance match. On the other hand, filters are normally considered as components which control the amplitude performance of a signal while maintaining a good impedance match. Looking at many system applications, however, group delay (and therefore phase behavior as a function of frequency) plays an important role as well – there are even special filter design procedures strongly concentrating on group delay/phase behavior (e.g. Bessel or linear phase filters). This indicates closer relations between the two concepts than usually perceived.

In this paper, we deepen and extend the discussion in [12], and compare the design of CRLH transmission lines with the design of high-order Chebyshev filters in more detail. It is proven that a finite length CRLH transmission line can be considered as the central part of a Chebyshev filter. Thus, if we design a CRLH transmission line based on the classical filter theory, a better performance of a finite length transmission line section can be achieved. To this end, we firstly summarize the characteristic formulas of a CRLH transmission line. Secondly, we recall the design formulas for a Chebyshev band-pass filter with a similar characteristic and prove that the central section of the Chebyshev band-pass filter is identical to a finite section of a CRLH transmission line. By attaching the ending sections of the filter, a finite length CRLH transmission line with better performance is achieved. A similar result can be obtained for dual-CRLH transmission lines. Thirdly, we compare the group delay and phase distribution between a CRLH transmission line and a Chebyshev filter. Backward wave propagation inside a Chebyshev filter is indicated. Finally, we present an example on how to improve the design of CRLH transmissions lines in a simple way.

II. CRLH TRANSMISSION LINE IN THE BALANCED CASE

The equivalent circuit of a CRLH transmission line is a cascaded network as shown in Fig. 1(a) which consists of series LC resonators alternating with shunt LC resonators. It is a periodic structure, and the LC components are

$$\begin{cases} L_j = L_R, & C_j = C_R, & \text{for series branch resonators} \\ L_k = L_L, & C_k = C_L, & \text{for shunt branch resonators} \end{cases} \quad (1)$$

where the subscripts L and R denote the “left handed” and the “right handed” case, respectively. The resonant frequencies of the left/right handed LC circuits are $\omega_L = \frac{1}{\sqrt{L_L C_L}}$ and

$\omega_R = \frac{1}{\sqrt{L_R C_R}}$. The resonant frequencies of the series-branch

resonators and shunt-branch resonators are $\omega_{se} = \frac{1}{\sqrt{L_R C_L}}$

and $\omega_{sh} = \frac{1}{\sqrt{L_L C_R}}$. We define one series LC resonator $L_j C_j$

together with one adjacent shunt LC resonator $L_k C_k$ as a unit cell to analyze the periodic structure. The characteristic impedance Z_C of the CRLH transmission line obtained from the image impedance analysis [14] is

$$Z_C = \sqrt{Z_L Z_R \frac{\varepsilon_{se}}{\varepsilon_{sh}} \left(1 - \frac{\varepsilon_{se} \varepsilon_{sh}}{4} \frac{\omega_L}{\omega_R} \right)} \quad (2)$$

where $\varepsilon_{se} = \frac{\omega}{\omega_{se}} - \frac{\omega_{se}}{\omega}$, $\varepsilon_{sh} = \frac{\omega}{\omega_{sh}} - \frac{\omega_{sh}}{\omega}$, $Z_L = \sqrt{\frac{L_L}{C_L}}$ and

$Z_R = \sqrt{\frac{L_R}{C_R}}$. If $\varepsilon_{se} = \varepsilon_{sh}$ holds (equivalent to $\omega_{se} = \omega_{sh}$), the

band gap between the left/right handed regions vanishes. This case is often referred to the balanced case of CRLH transmission lines. A comparison of the characteristic impedance of a CRLH transmission line between the balanced and unbalanced case is shown in Fig. 2. Even in the balanced case, a perfect impedance match can be only achieved at the transition frequency ω_0 .

The balanced condition of a CRLH transmission line is

$$\omega_{se} = \omega_{sh} = \omega_0 \quad \text{or} \quad L_L C_R = L_R C_L \quad (3)$$

where ω_0 is the transition angular frequency between the left/right handed regions. The same results can be found in [5] and [6].

In the balanced case, the characteristic impedance of the CRLH transmission line is

$$Z_C = Z_0 \sqrt{1 - \frac{\varepsilon^2 \omega_L}{4 \omega_R}} \quad (4)$$

where $\varepsilon = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$ and $Z_0 = Z_L = Z_R$. Assuming an

infinitely long CRLH transmission line in the balanced case, the pass-band of the CRLH transmission line is corresponding to $Z_C > 0$. Thus, the lower and upper cut-off frequencies are determined by the condition $Z_C = 0$. We obtain the corresponding equation on ω from (4) as

$$\frac{1}{4} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^2 \frac{\omega_L}{\omega_R} = 1 \quad (5)$$

Then, we obtain the lower and higher cut-off frequencies by solving (5) as

$$\begin{cases} \omega_{cL} = \omega_R \left(\sqrt{1 + \frac{\omega_L}{\omega_R}} - 1 \right) \\ \omega_{cR} = \omega_R \left(\sqrt{1 + \frac{\omega_L}{\omega_R}} + 1 \right) \end{cases} \quad (6)$$

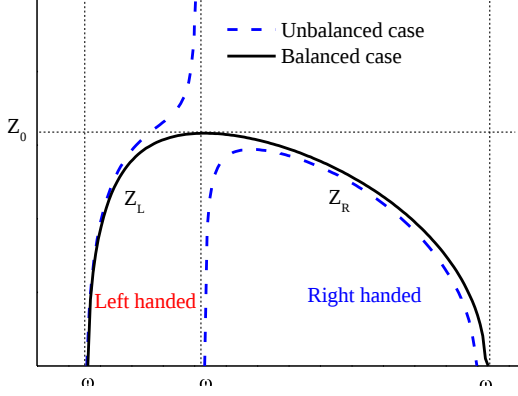


Fig. 2 Characteristic impedance of a CRLH transmission line in the balanced/unbalanced case

From (3) and (5), the relation between the transition frequency and cut-off frequencies is

$$\omega_0^2 = \omega_{cL} \omega_{cR} = \omega_{sh} \omega_{se} = \frac{1}{\sqrt{L_L L_R C_L C_R}} \quad (7)$$

There are four independent parameters, namely L_L , C_L , L_R , and C_R , in a CRLH transmission line design. With the balanced condition, only three independent parameters can be chosen freely. Once two cut-off frequencies and the characteristic impedance Z_C (matching to Z_0) are fixed, a unique CRLH transmission line configuration is determined by (5) and (6) by subtracting the two equations in (5) and introducing Z_0 and ω_0

$$\begin{cases} \omega_0 L_R = \frac{1}{\omega_0 C_L} = \frac{\omega_0}{\omega_{cR} - \omega_{cL}} 2Z_0 \\ \omega_0 C_R = \frac{1}{\omega_0 L_L} = \frac{\omega_0}{\omega_{cR} - \omega_{cL}} \frac{2}{Z_0} \end{cases} \quad (8)$$

III. CHEBYSHEV BAND-PASS FILTER

An N^{th} order band-pass filter in principle has the same LC equivalent circuit as the CRLH transmission line in Fig. 1(a). The band-pass filter design is usually achieved from the low-pass to band-pass transformation, in which a prototype low-pass filter is applied. The mapping formulas can be found in [14] with

$$\begin{cases} \omega_0 L_j = \frac{1}{\omega_0 C_j} = \frac{\omega_0}{\omega_2 - \omega_1} g_j Z_0, \text{ for series branch resonators} \\ \omega_0 C_k = \frac{1}{\omega_0 L_k} = \frac{\omega_0}{\omega_2 - \omega_1} \frac{g_k}{Z_0}, \text{ for shunt branch resonators} \end{cases} \quad (9)$$

where g_i is the i^{th} element value of the prototype low-pass filter, ω_1 and ω_2 are the lower and upper cut-off frequencies, respectively, and $\omega_0 = \sqrt{\omega_1 \omega_2}$ is the center frequency. Z_0 is the system impedance. Once g_i , ω_1 , ω_2 and Z_0 are fixed, a band-pass filter is uniquely determined by (8).

From (8) we can obtain

$$L_j C_j = L_k C_k = \frac{1}{\omega_0^2} \quad (10)$$

which is equivalent to (3). In a band-pass filter design, the balanced case of a CRLH transmission line always holds.

Since the mapping from a prototype low-pass to a band-pass filter is a generic formula, it can be applied to any type of prototype low-pass filter. Thus, the balanced case of a CRLH transmission line is automatically realized in band-pass filters from *any kind* of prototype low-pass filters with series- LC and shunt- LC resonators. Butterworth, Gaussian, or Chebyshev (with any pass-band ripples) band-pass filters constructed from (8) satisfy the balanced condition of a CRLH transmission line.

In most prototype low-pass filters, the element values g_i usually vary in a certain range and lead to a non-periodic structure. The central section of a high order Chebyshev filter, however, has a periodic structure. Fig. 3 shows an example of a 41st order Chebyshev prototype low-pass filter with several values of pass-band ripple. For elements not close to either end, element values are practically periodic. It should be noticed that always two adjacent filter elements form one equivalent transmission line cell. Thus, the central part of a Chebyshev is a quasi-periodic structure, independently of the pass-band ripple.

A. Low Pass-Band Ripple

The element values of a 41st order Chebyshev prototype low-pass filters with several different values of pass-band ripple are shown in Fig. 3. The lower the pass-band ripple, the lower is the variation of the element values. In the limiting case, the values of the central elements are close to $g_i=2$. When the pass-band ripple in a Chebyshev low-pass filter is low and its order is high, it can be proven theoretically or shown numerically that the filter element values approach $g_i=2$ for all elements that are not close to either filter end.

When the element value is $g_j=g_k=2$, (7) and (8) become equivalent to each other. Thus, if a Chebyshev band-pass filter and a CRLH transmission line have the same cut-off frequencies (i.e. $\omega_1=\omega_{cL}$, $\omega_2=\omega_{cR}$) and system impedance Z_0 , the same LC equivalent circuit is determined by either (7) or (8). Once ω_1 , ω_2 and Z_0 are fixed, the corresponding CRLH transmission line in the balanced case and the central part of the corresponding high order Chebyshev band-pass filter with low pass-band ripple have identical LC configurations.

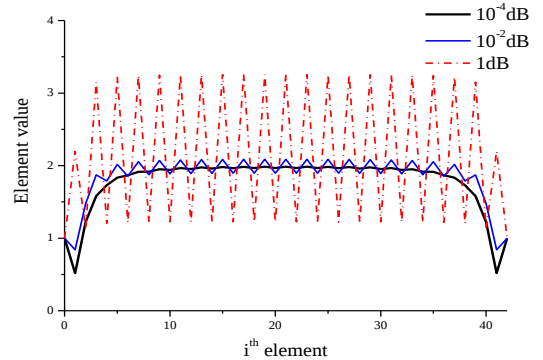


Fig. 3 Element values of 41st order prototype Chebyshev low-pass filters with different pass-band ripples

B. High Pass-Band Ripple

In a high-order Chebyshev prototype low-pass filter with larger pass-band ripple, element values oscillate around $g = 2$ as shown in Fig. 3. For those central elements, element values of series-branch LC resonators and shunt-branch LC resonators keep constant with g_{se} and g_{sh} , alternatively, and $g_{se} \cdot g_{sh} \approx 4$ always holds. Thus, a high-ripple high-order Chebyshev band-pass filter has a periodic configuration as well. It is a modified balanced CRLH transmission line which will be discussed in section IV.

Once two cut-off frequencies and the system impedance are given, a uniform CRLH transmission line in the balanced case and a high order Chebyshev band-pass filter with low pass-band ripple can be uniquely implemented with LC circuits, respectively. In other words, a finite length uniform CRLH transmission line in the balanced case is equivalent to the central part of a high-order low-ripple Chebyshev band-pass filter with the same cut-off frequencies and system impedance.

IV. IMPEDANCE MATCH AND PASS-BAND RIPPLE

We first consider a CRLH transmission line in the balanced case. The characteristic impedance is dependent on frequency as in (4), and the calculated results are shown in Fig. 2. A match between the CRLH transmission line and the system impedance Z_0 can be obtained only around the transition frequency ω_0 . This indicates that impedance matching circuits should be applied in broadband designs and applications.

When the pass-band ripple of a Chebyshev prototype low-pass filter is high, the element values of the central part are alternatively $g_{se} = 2\alpha^2$ and $g_{sh} = \frac{2}{\alpha^2}$ for series and shunt branches, where α is a coefficient dependent on the pass-band ripple. If Z_0 in (7) is replaced by αZ_0 , it is still a well designed CRLH transmission line. Therefore, the pass-band ripple of a Chebyshev band-pass filter corresponds to an impedance mismatch between the applied and designed system impedance of the CRLH transmission line. The greater the difference, the higher is the pass-band ripple. Based on the analysis of element values of a high-order Chebyshev prototype low-pass filter, the relation between the pass-band ripple and the impedance mismatch is

$$\alpha = \left(\frac{g_{se}}{g_{sh}} \right)^{\frac{s}{2}} = \coth^s \left(\frac{1}{2} \sinh^{-1} \left(10^{\frac{L_{Ar}}{10}} - 1 \right)^{-\frac{1}{2}} \right) \quad (11)$$

where L_{Ar} is the pass-band ripple in dB, and the sign function is $s = \begin{cases} +1, & Z_0^{Designed} > Z_0^{Applied} \\ -1, & Z_0^{Designed} < Z_0^{Applied} \end{cases}$. Once α is given, the

pass-band ripple can be obtained from (10). For example, if the external system impedance is 50Ω and the characteristic impedance of a CRLH transmission line 70Ω , the

corresponding pass-band ripple of the band-pass filter is $L_{Ar} = 0.4827 \text{ dB} \approx 0.5 \text{ dB}$.

Therefore, the central part of a high-ripple high-order Chebyshev band-pass filter is equivalent to a CRLH transmission line with mismatched impedance to the source and load.

When the external and the design impedances of a CRLH transmission line are different, matching circuits have to be applied. Even if both impedances are equal, some matching may be required to improve the overall pass-band performance. Since a CRLH transmission line is equivalent to the central part of a Chebyshev band-pass filter, the impedance matching circuits can be constructed from the corresponding band-pass filter design. This procedure is equivalent to the design of an entire filter and automatically results in good matching performance.

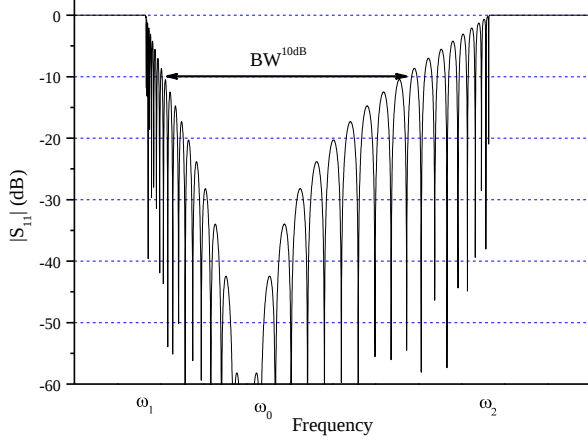
As an example, we have designed a CRLH transmission line of finite length in the balanced case, e.g. with 21 series resonators and 20 shunt resonators according to the circuit model in Fig. 1(a). The CRLH transmission line is composed of symmetrical unit elements [5], and the simulated voltage reflection coefficients are shown in Fig. 4(a). At the center frequency, the impedance match is almost perfect. This is the simplest method; nevertheless large ripples exist near the cut-off frequencies, which considerably affect the pass-band of the CRLH transmission line. On the other hand, if we design the CRLH transmission line from a 41st order Chebyshev filter with 0.01 dB pass-band ripple by the method in Section III, the impedance match is excellent over the whole pass-band as shown in Fig. 4(b). However, all 41 resonators have to be designed exactly using the required element values.

The trade-off is a hybrid design, in which the periodic structure of the CRLH transmission line is applied, but several resonators near either end are exchanged with those from the corresponding band-pass filter, e.g. a 41th order Chebyshev band-pass filter with 0.01 dB pass-band ripple. The detailed procedure for our example is: a) design a 41st order Chebyshev filter with 0.01 dB pass-band ripple and a CRLH transmission line with 41 periodic unit cells with the same cut-off frequencies and characteristic impedance, respectively; b) choose the first and last three LC resonators from the Chebyshev filter, and place them into the CRLH transmission line at the corresponding positions. The simulated results of the hybrid design with three resonators exchanged at either end are shown in Fig. 4(c). A better impedance match compared to Fig. 4(a) has been achieved. With the modification of more resonators, better results can be obtained.

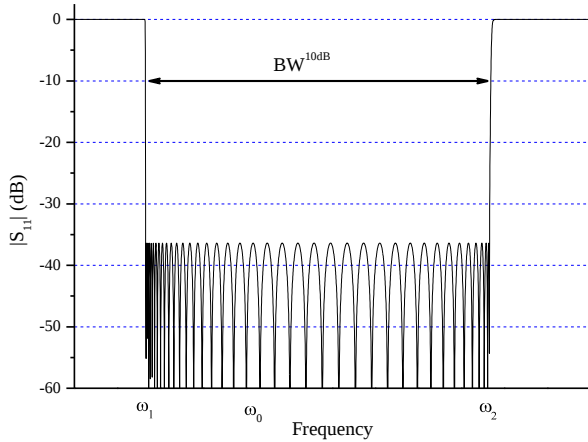
If the dual-CRLH transmission line is taken into consideration, a similar procedure can be applied. The pass-band ripple of a high-order Chebyshev band-stop filter corresponds to the mismatch between the characteristic impedance of a dual-CRLH transmission line and the system impedance.

In the case that even order Chebyshev filters are concerned, the load impedance, which is only dependent on the pass-band ripple and independent on the filter order, is always different from the source impedance. However, when

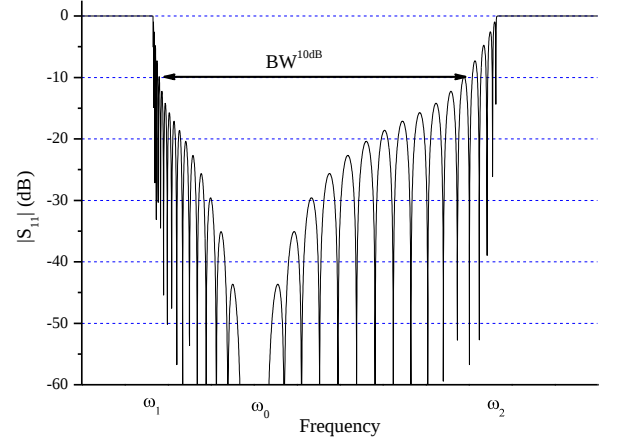
the pass-band ripple is small, the load impedance is close to the source impedance. Therefore, there is not much difference between filter element values of even and odd order for low pass-band ripple and high order of the filter. On the other hand, when the pass-band ripple is high, the difference between even order and odd filters appears only in a limited number of elements near the load; these can be understood as an impedance matching circuit. Thus, with respect to even order Chebyshev filters, the analysis in this paper is also suitable.



(a)



(b)



(c)

Fig. 4 Comparison of 10 dB bandwidths between CRLH transmission lines. (a) Conventional CRLH transmission line with 20 symmetric unit elements; (b) CRLH transmission line from 41th order Chebyshev band-pass filter with 0.01 dB pass-band ripple; (c) CRLH transmission line with three resonators exchanged at either end.

Another point that we would like to stress here is the design flexibility of a CRLH transmission line. Between CRLH transmission lines and Chebyshev band-pass filters, there are three common parameters, i.e. two cut-off frequencies and the matching impedance. Besides these parameters, there is one more parameter in the design of band-pass filters, i.e. the pass-band ripple. It reflects the impedance mismatch between CRLH transmission lines and source/load. Once some pass-band ripple variation is allowed in a limited range, it gives more flexibility in the design of a CRLH transmission line.

V. GROUP DELAY AND NEGATIVE PHASE VELOCITY

Left-handed transmission lines support electromagnetic waves with phase and group velocities that are antiparallel to each other. The apparent backward wave propagation is one key characteristic of CRLH transmission lines. We will show that this does exist with Chebyshev band-pass filters in part of the frequency band due to a similar phase performance compared to CRLH transmission lines.

Group delay, which is an important parameter of a filter, is defined as

$$t_g = -\frac{d\phi}{d\omega} \quad (12)$$

Flat and consistent group delay over frequency is of importance in many microwave systems. A comparison of group delay between a conventional finite length CRLH transmission line, the corresponding Chebyshev band-pass filter, and the modified CRLH transmission line is shown in Fig. 5. They all have a similar performance over frequency. There are no ripples in the case of the band-pass filter (Fig. 5(b)), while ripples obviously exist around the cut-off frequencies ω_1 and ω_2 in Fig 5(a). The performance of the new designed CRLH transmission line, as shown in Fig. 5(c), lies between the conventional CRLH transmission line and the Chebyshev filter. As can be seen, Chebyshev filters have a

better group delay performance compared to conventional, finite length CRLH transmission lines.

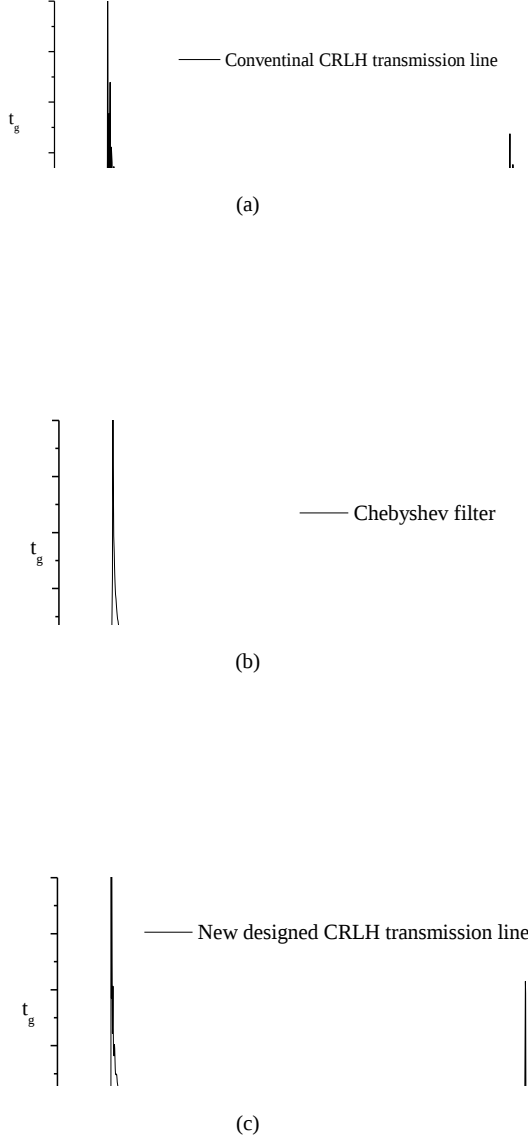


Fig. 5 Group delay of a finite length CRLH transmission line with 20 symmetric unit elements, the correspondent 41th order Chebyshev band-pass filter, and the modified CRLH transmission line with three resonators from the Chebyshev band-pass filter at either end

When a band-pass filter is built from lumped/quasi-lumped elements, the dimensions of each element are much smaller than the wavelength in the pass-band of the filter. The phase shift along the filter can be calculated based on its equivalent *LC* circuit. With the example of a 41th order Chebyshev band-pass filter with 0.01 dB pass-band ripple, there are twenty nodes (the connection nodes between series-branch and shunt-branch resonators) in the filter. The phase shifts along

those nodes at different frequencies are shown in Fig. 6. Similar results are obtained for a conventional CRLH transmission line. At the center frequency ω_0 , all nodes exhibit the same phase, which is known as the zeroth order resonance in a conventional CRLH transmission line. In the right-handed region, phase angles decrease along the nodes according to a forward wave propagation. In the left-handed region, phase angles increase along the nodes, approximating a negative phase velocity.

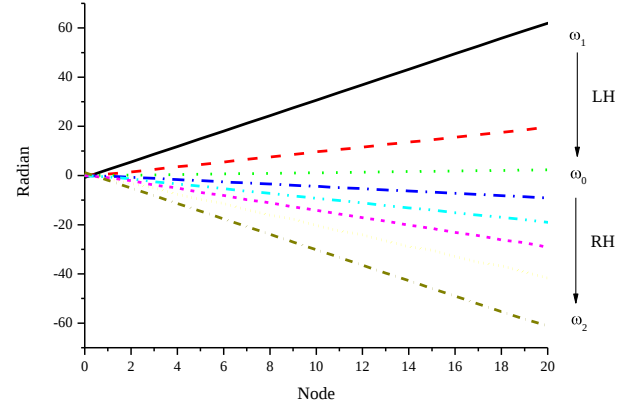


Fig. 6 Phase shifts along a 41st order Chebyshev band-pass filter from the lower cut-off frequency ω_1 to the higher cut-off frequency ω_2 . The left-handed (LH) region is between ω_1 and ω_0 , and the right-handed (RH) region is between ω_0 and ω_2 .

This phase analysis shows the equivalence between a finite length transmission line and a Chebyshev filter also with respect to approaching a negative phase velocity, where the filter has a superior amplitude and group delay performance compared to conventional finite length CRLH transmission lines.

VI. CONCLUSIONS

The characteristics of finite length CRLH transmission lines and high order Chebyshev band-pass filters are analyzed, and synthesis formulas based on the matching impedance and cut-off frequencies are shown. From the analysis of element values in Chebyshev prototype low-pass filter, close relations between a CRLH transmission line and a Chebyshev band-pass filter are revealed. It is proven that a CRLH transmission line - in the balanced case - can be considered as the central part of a high-order low pass-band ripple Chebyshev band-pass filter with identical matching impedance and cut-off frequencies.

The meaning of the pass-band ripple in a Chebyshev band-pass, which has no obvious counterpart in a CRLH transmission line, corresponds to an impedance mismatch between the CRLH transmission line and the source/load. The formula to compute the pass-band ripple from the impedance mismatch is presented as well. In addition, impedance matching circuits in finite length CRLH transmission line applications can be design based on the classical filter theory, achieving much better performance.

Phase analysis, including group delay and phase distribution, shows that Chebyshev filters have similar and partly even better phase responses compared to conventional finite length CRLH transmission lines. Negative phase velocity is supported by Chebyshev filters in the same way as with CRHL transmission lines. This confirms the relation between Chebyshev filters and CRLH transmission lines.

Similar results can be obtained for dual-CRLH transmission lines (another kind of negative refractive index transmission lines) and Chebyshev band-stop filters.

Summarizing, this means that a finite length CRLH/dual-CRLH transmission line can be designed from a Chebyshev band-pass/band-stop filter. By allowing a reasonable pass-band ripple, there is even more design freedom. Based on impedance matching, the design from classical filter theory can achieve smooth broadband responses.

First examples for such an approach have already been presented in [8] and [9] where sections of a CRLH transmission line have been realized as band-pass filters to design antennas with backfire-to-endfire scanning.

Finally, we would like to state that we do not at all want to replace the CRLH transmission line concept by filter theory. The CRLH transmission line concept is very useful to get new insight and to arrive at new design concepts, mainly by phase/dispersion engineering which is generally not at all evident from filter theory, but filter theory may help to improve its performance in the case of finite CRHL transmission line length.

APPENDIX

ELEMENT VALUES OF A HIGH ORDER PROTOTYPE CHEBYSHEV LOW-PASS FILTER

An N^{th} order prototype Chebyshev low-pass filter with a pass-band ripple of L_{Ar} dB, a cut-off frequency at $\omega'_0 = 1$ and a normalized impedance $g_0 = 1$, has the initial coefficients [14]

$$\begin{cases} \beta = 2 \sinh^{-1} \left(10^{\frac{L_{Ar}}{10}} - 1 \right)^{-\frac{1}{2}} \\ \gamma = \sinh \left(\frac{\beta}{2N} \right) \end{cases} \quad (13)$$

and the corresponding coefficients

$$\begin{cases} a_k = \sin \left[\left(k - \frac{1}{2} \right) \frac{\pi}{N} \right] \\ b_k = \gamma^2 + \sin^2 \left(k \frac{\pi}{N} \right) \end{cases} \quad (14)$$

The element values are $g_1 = 2 \frac{a_1}{\gamma}$ and $g_k = \frac{4a_{k-1}a_k}{b_{k-1}g_{k-1}}$, where $k = 2, 3, \dots, N$. Assuming that the filter order is high, i.e. $N \gg \max(1, \beta, \gamma)$ approaches $\frac{\beta}{2N}$.

In the beginning part of the prototype low-pass filter ($k \ll N$), the corresponding coefficients are

$$\begin{cases} a_k ; \frac{2k-1}{2N} \pi \\ b_k ; \left(\frac{\beta}{2N} \right)^2 + \left(\frac{k\pi}{N} \right)^2 \end{cases} \quad (15)$$

Then the element values of the prototype low-pass filter are

$$\begin{aligned} g_{2k+1} &= j \frac{4k+1}{\beta'} \pi \frac{\prod_{m=1}^k \left(1 - \frac{\beta'^2}{2m-1} \pi^2 \right)}{\prod_{m=1}^k \left(1 - \frac{\beta'^2}{2m} \pi^2 \right)} \prod_{m=1}^k \left(\frac{2m-1}{2m} \right) \\ g_{2k+2} &= j \frac{4k+3}{k-1} \beta' \pi \frac{\prod_{m=1}^k \left(1 - \frac{\beta'^2}{2m} \pi^2 \right)}{\prod_{m=1}^{k+1} \left(1 - \frac{\beta'^2}{2m-1} \pi^2 \right)} \prod_{m=1}^k \left(\frac{2m}{2m-1} \right) \end{aligned} \quad (16)$$

where $\beta' = j \frac{\beta}{2}$ and $k = 1, 2, 3, \dots, k \ll N$.

The element values can be approximated by the infinite products of the sine and cosine functions and the Wallis' series. Then the element values are

$$\begin{cases} g_{2k+1} \approx j \frac{2}{2k+1} \frac{4k+1}{\sin \beta'} \frac{\cos^2 \frac{\beta'}{2}}{\frac{\beta'}{2}} \approx j \frac{4 \cos^2 \frac{\beta'}{2}}{\sin \beta'} \\ g_{2k+2} \approx -j \frac{2k}{2} \frac{4k+3}{2k+1} \frac{\sin \beta'}{\cos^2 \frac{\beta'}{2}} \approx -j \frac{\sin \beta'}{\cos^2 \frac{\beta'}{2}} \end{cases} \quad (17)$$

Substituting β' by $j \frac{\beta}{2}$, the element values can be rewritten as

$$\begin{cases} g_{\text{odd}} = g_{2k+1} \approx 2 \coth \frac{\beta}{4} = 2\alpha \\ g_{\text{even}} = g_{2k+2} \approx 2 \tanh \frac{\beta}{4} = \frac{2}{\alpha} \end{cases} \quad (18)$$

where $\alpha = \coth \frac{\beta}{4}$. β is dependent on the pass-band ripple L_{Ar} dB as in (12). When L_{Ar} is small, we have $\beta \ll 1$.

Therefore, for a high-order low-ripple ($N \gg \beta \gg 1$) prototype Chebyshev low-pass filters, we have

$$g_{\text{odd}} ; g_{\text{even}} ; 2 \quad (19)$$

The element values approach 2 after the beginning parts of the filter.

On the other hand, when the pass-band ripple is high, i.e. $\beta \approx 1$ or $\beta < 1$, (18) shows that the element value of g_i will switch between $g_{\text{odd}} > 2$ and $g_{\text{even}} < 2$. The adjacent element values satisfy $g_{\text{odd}} \cdot g_{\text{even}} ; 4$.

For a high-order Chebyshev prototype low-pass filter, i.e. $N \gg \max(1, \beta)$, a general relation between two adjacent element values is obtained from (15)

$$g_{2k+1} = \frac{4}{g_{2k}} \cdot \frac{a_{2k} a_{2k+1}}{b_{2k}} \approx \frac{4}{g_{2k}} \cdot \frac{\sin \frac{4k+1}{2N} \pi \sin \frac{4k+3}{2N} \pi}{\sin^2 2k \frac{\pi}{N}} \quad (20)$$

When the element is not close to either end of the filter, i.e. $k \gg 1$ and $N - k \gg 1$, we have $g_{2k} \cdot g_{2k+1} \approx 4$.

Thus the sequence of element values keeps $g_i \approx 2$ of a high-order ($N \gg \max(1, \beta)$) Chebyshev prototype low-pass filter with low pass-band ripple ($L_{Ar} = 1$), when those elements are not close to either end of the filter. Otherwise, when the pass-band ripple of the filter is high, those element values will keep switching between $g_{odd} > 2$ and $g_{even} < 2$.

For example, we show (Fig. 3) some calculated results of the element values of Chebyshev low-pass filters. When $N > 20$, $4 < k < N - 4$ and $L_{Ar} = 0.01$ dB, the maximum relative error of the element values to $g = 2$ is less than 8%. If the filter order goes higher and the pass-band ripple becomes lower, e.g. $N > 30$ and $L_{Ar} = 0.001$ dB, those errors are less than 6%. Moreover, those errors for elements around the central part of the filter keep always below 3%.

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Changjun Liu received the M.S. degree and Ph.D. degree from Sichuan University in China, in 1997 and 2000, respectively. From 2001 to 2002, he was a Post-Doctoral Researcher with the Seoul National University, Seoul, Korea. Since July 2004 he is a professor with the School of Electronics and Information Engineering, Sichuan University. He was a visiting scientist with the Institute of Microwave Techniques, University of Ulm, Germany from Dec. 2006 to Dec. 2007. He is a senior member of IEEE. His research interests are advanced microwave components using metamaterials.

Wolfgang Menzel received the Dipl.-Ing. degree in electrical engineering from the Technical University of Aachen, Aachen, Germany, in 1974, and the Dr.-Ing. degree from the University of Duisburg, Duisburg, Germany, in 1977. From 1979 to 1989, he was with the Millimeter-Wave Department, AEG, Ulm, Germany [now the European Aerospace, Defense, and Space Systems (EADS)]. From 1980 to 1985, he was Head of the Laboratory for Integrated Millimeter-Wave Circuits. From 1985 to 1989, he was Head of the entire Millimeter-Wave Department. During that time, his areas of interest included planar integrated circuits (mainly on the basis of finline techniques), planar antennas, and systems in the millimeter-wave frequency range. In 1989, he became a Full Professor with the Institute of Microwave Techniques, University of Ulm, Ulm, Germany. His current areas of interest are multilayer planar circuits, waveguide filters and components, antennas, millimeter-wave and microwave interconnects and packaging, and millimeter-wave application and system aspects.

Dr. Menzel was an associate editor for the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES (2003-2005). From 1997 to 1999, he was a Distinguished Microwave Lecturer for Microwave/Millimeter Wave Packaging. From 1997 to 2001, he chaired the German IEEE Microwave Theory and Techniques (MTT)/Antennas and Propagation (AP) Chapter. He was the recipient of the 2002 European Microwave Prize.