Design of Experiment for the Characterization of a 160 GHz Radar MMIC

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***Abstract*—In this paper, the methods of design of experiment (DoE) are applied to evaluate a 160 GHz radar monolithic microwave integrated circuit (MMIC). Due to a strong nonlinear transmitter (Tx) with five adjustable input variables and several cascaded circuits, the search for maximum RF output power by varying the parameters successively is not a promising approach. With DoE the input parameter space is optimally sampled to maximize the gain of information with a minimum of required measurement points. By the analysis of variance (ANOVA) a multidimensional polynomial fit of the measured data is stepwise optimized, which results in a lower standard deviation between model and measurement data and higher predictability. Test measurements confirm the model of the DoE approach.**

***Index Terms*—Design of experiment, DoE, ANOVA, MMIC, Millimeter wave, Radar, FMCW.**

I. Introduction

The increase of transistor oscillation frequencies f_{max} up to 500 GHz in low-cost semiconductor technologies, like Silicon-Germanium (SiGe) and CMOS, enables many communication and radar sensor applications in the mm-wave range [1]. Typically, the front-ends are completely integrated into an MMIC, which results in simpler low frequency off-chip interconnects, but at the prize of a higher MMIC complexity. Break-outs of circuits need additional expensive chip area and are often difficult to measure due to differential circuits and interstage matching. Although the MMIC design process and optimization is mature in many simulation tools, accurate measurements are necessary to proof the simulated behavior.

The first measurement approach of such complex systems is to successively vary the input parameters around one operating point and measure the output parameters. This leads to a very limited knowledge of the investigated system because the interactions between parameters are completely unobserved. The second approach to test all input parameter combinations would increase the number of single measurements enormously. For a system with five input parameters, each set to ten different levels would result in 10^5 single measurements. Assuming a measurement time of 30 seconds this would result in a measurement time of over one month without any post processing. This conflict between high gain of information and low measurement time is well-known. A very good trade-off between both approaches is the design of experiment.

In this paper, the Tx-side of a 160 GHz radar MMIC is characterized with the methods of DoE to illustrate its power for the evaluation of strong nonlinear frequency converting circuits. The concept of this evaluation and the required steps are explained. Based on the block diagram of the Tx-side of the radar MMIC, the definitions of inputs and outputs are given in Section II. Subsequently, a simplified test called screening plan is conducted to gain a first measure of the influences of each input parameter on the RF output power. Based on these results the design and analysis of the main experiment is discussed in Sections IV and V, in which the model-based approach of DoE is illustrated in detail. Finally, the DoE results are verified by test measurements in Section VI.

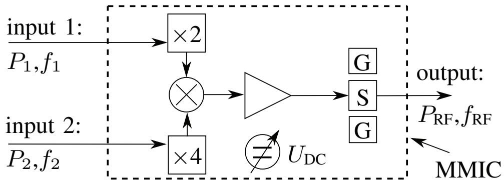

Fig. 1. Block diagram of the Tx-side of the radar MMIC with two inputs and the RF output at the GSG probe pad.

TABLE I
Properties of the Input Parameters (Factors)

Symbol	Parameter (= Factor)	Unit	Values min.	center	max.
A	f_1	GHz	57.6	58.6	59.6
B	P_1	dBm	−2.0	1.0	4.0
C	f_2	GHz	8.0	10.5	13.0
D	P_2	dBm	−25.0	−12.5	0.0
E	U_{DC}	V	2.8	3.0	3.2

II. 160 GHz Radar MMIC and its Interfaces

The schematic of the integrated radar transmitter under test is depicted in Fig. 1. Input 1 is the fixed-frequency local oscillator (LO) input with comparably low phase noise in the radar mode, whose frequency f_1 and power P_1 are the first two input parameters or factors [2], [3]. The frequency ramp signal for the frequency modulated continuous wave (FMCW) radar is fed at input 2 to the MMIC. Its power P_2 and frequency f_2 are two further factors. The frequency-

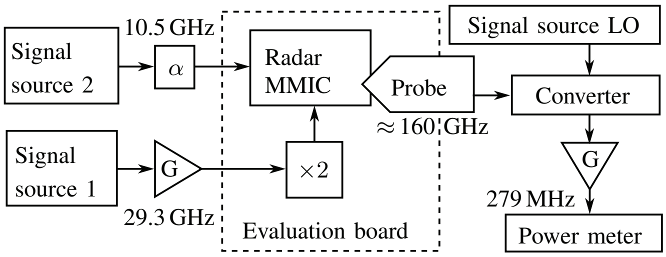

Fig. 2. Schematic view of the MMIC measurement setup.

quadrupled ramp is up converted by the mixer with the doubled input 1 signal ($4 \times f_2 + 2 \times f_1$). Finally, the RF signal is amplified and guided to a Ground-Signal-Ground (GSG) pad. By mixing the ramp signal with the low phase noise LO signal to RF, the multiplication factor of the ramp signal is reduced compared to an RF ramp signal generation based solely on multipliers, which results in an improved RF phase noise. The supply voltage U_{DC} of the circuits completes the factor list in Tab. I. Additionally, the temperature is a disturbance variable, which is fixed to 30 °C for all measurements. The goal of the subsequent investigation is to find a polynomial function of the factors to predict the RF output power P_{RF} and its maximum.

III. Factor Screening as Preliminary Study

After the factor definitions, the first step of the DoE is the factor screening [2] to get a first estimate of the influence of the factors on the RF output power. Typically a linear approximation is used, shown as polynomial of first order exemplary for three factors in (1).

$$P_{\mathrm{RF}} = c_0 + c_A x_A + c_B x_B + c_C x_C + c_{AB} x_A x_B + c_{AC} x_A x_C + c_{BC} x_B x_C + c_{ABC} x_A x_B x_C. \quad (1)$$

c_0 represents the mean of all measured RF powers levels. The main effects are indicated with c_A to c_C each depending only on one factor x_i. The interactions are indicated with c_{AB} to c_{ABC}. A dependency of two variables like c_{AB} is called two-way interaction, a dependency of three variables like c_{ABC} is called three-way interaction and so on. This system of equations can be completely solved with eight single measurements. In the present case with five factors 32 single measurements would be necessary, in which every factor is set to its minimum and maximum value. A reduction to 16 single measurements leads to confounding of effects because of the under-determined system of equations. For instance, the main effect c_A is confounded with the four-way interaction c_{BCDE}. Details about confounding of effects are explained in [2]. Three-way and higher order interactions are typically neglected compared to main effects and two-way interactions in the screening plan [3]. Consequently, a factor screening plan with 16 single measurements is applied.

The measurement setup for the data acquisition is depicted in Fig. 2. Signal source 1 is amplified with an external amplifier, frequency-doubled with a doubler chip on the evaluation board, and fed to the radar MMIC. To provide the required power range for the other input, a fixed attenuator α is inserted between signal source 2 and the radar MMIC. The RF signal is guided from the MMIC via the probe tip to the external converter module, where the RF signal is down converted with the signal source LO to 279 MHz and fed into the power meter. The characteristics of the doubler chip and the converter module were determined in preliminary measurements for a deembedding as accurate as possible.

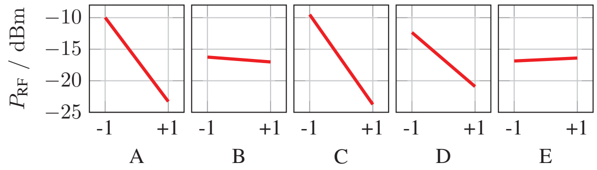

Fig. 3. Main effects of the five factors on a normalized x-axis.

TABLE II
Number of Required Measurement Points

n_{factor} \ n_{order}	1	2	3	4	5
1	2	3	4	5	6
2	3	6	10	15	21
3	4	10	20	35	56
4	5	15	35	70	126
5	6	21	56	126	252

The mean at the minimum (normalized −1) and maximum (normalized +1) factor value are shown in Fig. 3 for all factors. The respective main effect is calculated as the difference of the two means. The factors B and E have a comparably low main effect on the output power. Factor B, the power of the high frequency input P_1, was expected to have a stronger impact. The other three factors show an expected comparably strong impact on the RF output power level of the MMIC.

IV. Selection of the Polynomial Model

After the factor screening, the main part of the DoE is to choose a suitable model for the system under test and to conduct the measurements and analyze the data. The goal of the DoE is to generate a descriptive model of the MMIC, which does not necessarily explain the physical behavior but quantifies influences from the factors to the RF power level. Typically, this is achieved by a polynomial, whose order is of crucial importance. The higher the order, the better the model fit and the smaller the model errors. On the other hand a larger number of measurement points is required for modeling. The number of measurement points increases drastically with the model order n_{order} and the number of factors n_{factor}, as depicted in Tab. II. Consequently, a trade-off between the number of input parameters, the model order, and the model fit regarding standard deviation to the measured values has to be found. From the system's point of view, interactions between the two input powers and the frequencies

TABLE III
RESULTS OF THE ANOVA

	Full polyn.	Reduced polyn.
Sum of squares for res. variance	86.7	104.4
DF of residual variance	30	48
Mean squares of residuals	2.9	2.2
Standard deviation	1.7 dB	1.5 dB
R^2	98.7%	98.4%
Adjusted R^2	95.7%	96.7%
Prediction R^2	39.5%	90.3%

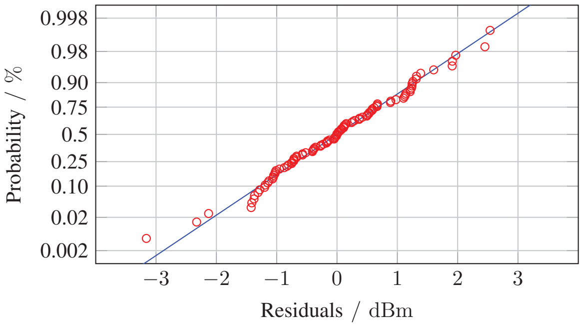

Fig. 4. Normal probability plot of the residuals of the model (○ ○) with a reference line (——) of ideally normally distributed residuals.

are expected in the model of the RF output power. In contrast, the interactions of the supply voltage with the other factors is assumed to be negligible. Thus, a reasonable reduction of factors is possible by fixing the supply voltage to the nominal value. This reduction is also confirmed by the small main effect of the supply voltage U_{DC}, (E) in Fig. 3.

To determine the model order, the expected system behavior and the possible measurement effort are critical. As a cascade of active circuit elements including two frequency multipliers and a mixer is characterized, a strong nonlinear behavior is expected. For that reason at least a second order model is required, which corresponds to a quadratic approximation. In many cases, where the relative value range of the parameters is below 10%, a model of second order can be sufficient. In this study, the relative parameter ranges are large and therefore, the model order should be increased. The main decision is between third (cubic) and fourth (quartic) order, because fifth and higher orders would lead to a huge number of measurement points. One advantage of fourth order polynomials is the good fitting of trough-shaped and wavelike curves compared to third order polynomials. For this reason and because 70 measurement points (see Tab. II) are still a reasonable quantity, the fourth order design was chosen for this investigation.

Additionally, five lack-of-fit points were added, which are not used for the model calculation, but to estimate the error of the model. Five important points for the model are repeated to quantify drifts of the output power over time. In the last step, another 20 points have been added to increase the robustness of the model by adding further redundancy for solving the system of equations. The required parameter settings for the 100 single measurements are determined by a statistics program.

V. Model Optimization and Evaluation

After the data acquisition the polynomial model is calculated from the measured points and optimized with the analysis of variance [2]. The ANOVA compares the strength of main effects and interactions to the strength of the remaining noise, which is quantified by the so-called p-value for each polynomial term c_i in (1). By stepwise removing insignificant terms of high p-value, the used full polynomial of fourth order is reduced. A comparison of the parameters for the full and the reduced polynomial is given in Tab. III. In the beginning, the 70 polynomial terms are determined from the 100 measurement points. This over-determined system of equations must have some residual variance, which is not explained by the model. The sum of squares for the residual variance of 86.7 is assigned to the model error and has 30 degrees of freedom (DF). The removal of insignificant polynomial terms increases the DF of the model error and the residual variance. Here, the reduced polynomial consists of 52 terms, resulting in a sum of squares for the residual variance of 104.4 and 48 DF for the residual variance. The ratio of the sum of squares for the residual variance and its degrees of freedom is called mean squares of residuals. This indicates the efficiency of the model and is noticeably improved from 2.9 to 2.2. By the term reduction, the standard deviation of the measurement points to the polynomial model is decreased from 1.7 dB to 1.5 dB. R^2, the part of the total variance, which is explained by the model, decreases monotonically with the reduction of terms, because every term explains a certain part of the total variance. The *adjusted* R^2 considers the number of model terms, so that it increases at the beginning of the term reduction process up to a local maximum, after which it declines again [2]. Here the *adjusted* R^2 was increased from 95.7% to 96.7%. Finally, the *prediction* R^2 indicates the fraction of variance that the model is expected to explain for a new data point [2]. The increase of 39.5% to 90.3% emphasizes a strong improvement of the predictability of the model by the reduction of polynomial terms based on the ANOVA.

To validate the generated model, the normal probability plot of the residuals, which are the difference between predicted and measured values, is shown in Fig. 4. This method visualizes, whether the residuals are normally distributed. The ideal case is depicted as a reference line (——). Here, the residuals are almost normally distributed, which confirms the correctness of the model.

The maximum output power was determined by evaluating the polynomial model with a multidimensional sweep and small grid. The largest calculated value was 3.4 dBm reached at f_1=58.9 GHz, P_1=-1.6 dBm, f_2=9.3 GHz, and P_2=-21.8 dBm. The RF output power for varying input frequencies f_1 and f_2 in Fig. 5 indicates the calculated maximum. The shift to lower ramp frequencies f_2 shows that the operating point of the amplifier after the mixer has to be corrected in

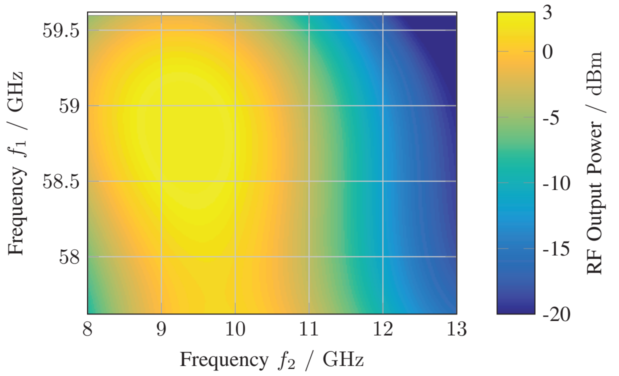

Fig. 5. Predicted RF output power P_{RF} at P_1=-1.6 dBm and P_2=-21.8 dBm for different input frequencies f_1 and f_2.

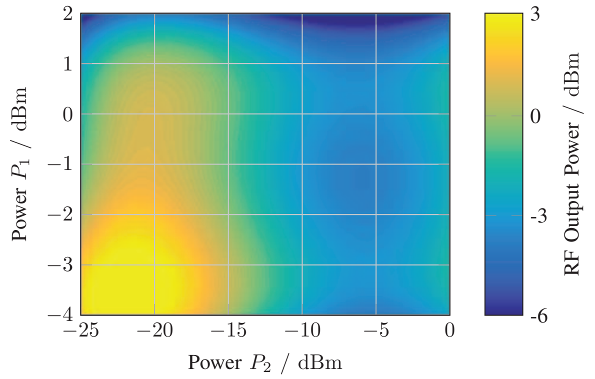

Fig. 6. Predicted RF output power P_{RF} at f_1=58.9 GHz and f_2=9.3 GHz for different input powers P_1 and P_2.

a redesign. The influence of the two input powers on the RF output power is lower than that of the input frequencies, depicted in Fig. 6. The decrease of RF output power for increasing input power P_2 stems from an integrated buffer amplifier, which already drives the subsequent quadrupler for low input powers P_2 into saturation. Additionally to the output value, the DoE also returns the standard deviation for every point in the parameter space. This simplifies the evaluation of the results. The standard deviation at the maximum RF output power is about 0.9 dB, which equals a probability of 95.4% that a measurement at this operating point would be between 1.6 dBm and 5.2 dBm.

VI. Model Verification by Test Measurements

The last step of the DoE procedure contains test measurements, whose values are compared to the predicted values from the polynomial model. Here, two cutting planes through the center of the parameter space are chosen. In the first one, the two input powers P_1 and P_2 are fixed and the RF output power is sampled at five different frequencies for f_1 and f_2 resulting in 25 measurement points. For the second cutting plane the powers are varied and the frequencies are fixed. The differences between measured and predicted output

TABLE IV
Difference between measured and predicted RF output power in dB for P_1=-1.0 dBm and P_2=-12.5 dBm (f_1,f_2 / GHz)

f_1 \ f_2	8.0	9.3	10.5	11.8	13.0
57.6	0.3	0.3	1.5	4.8	3.4
58.1	−0.7	−0.6	−0.7	2.0	1.1
58.6	−1.1	−1.7	−1.0	1.7	0.1
59.1	0.4	−1.2	−0.2	1.8	−0.1
59.6	3.1	0.8	2.1	4.1	1.4

TABLE V
Difference between measured and predicted RF output power in dB for f_1=58.6 GHz and f_2=10.5 GHz (P_1,P_2 / dBm)

P_1 \ P_2	−25.0	−18.8	−12.5	−6.3	0.0
−2.0	2.1	−2.7	−1.1	2.0	0.2
−0.5	0.9	−3.2	−1.6	1.5	−0.4
1.0	0.9	−2.1	−0.9	1.9	−0.3
2.5	−0.1	−1.7	−1.1	1.1	−1.6
4.0	3.1	3.0	2.6	4.0	0.6

powers are listed in Tabs. IV and V. Having in mind that the averaged standard deviation is 1.5 dB, the deviations are acceptable. The polynomial fit has its highest accuracy in the center of the parameter space. Consequently, deviations of up to 4 dB at the parameter limits are tolerable. To analyze of a specific region of the parameter space in more detail a second DoE is recommended. This design can contain less measurement points because a model of lower order might be sufficient to achieve the desired small standard deviation and high predictability in the smaller parameter space.

VII. Summary

In this paper, the techniques of DoE were applied for the characterization of the RF output power of an integrated radar transmitter at 160 GHz with five input parameters. A reasonable number of measurement points was determined to model the complex system behavior with a multidimensional polynomial of fourth order. With the ANOVA the full polynomial was stepwise reduced by insignificant terms resulting in an improved standard deviation of 1.5 dB and a *prediction* R^2 above 90%. The verification measurements proofed that the methods of DoE can create a polynomial model of the strong nonlinear radar transmitter with satisfactory accuracy from a low number of measurement points.

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