1 Introduction

1.1 Need for modulation

In electrical communications engineering the given task is to transmit signals which are available in electric shape (signal = physical demonstration of a message); the signals represent for instance speech, music, data. The preceding conversion into electric signals is done by the help of appropriate converters, for example a microphone.

When the signals are transmitted without a frequency conversion, it is called baseband transmission. The simplest example for it is a microphone with an amplifier and a loudspeaker. At the latest, disadvantages become obvious at the time when several signals should be transmitted simultaneously (every message channel needs its own transmission line) or if long distances need to be bridged (practically, low-frequency signals cannot be sent and received by antennas because the dimensions of an antenna always need to be in the order of magnitude of the wavelength).

Therefore, the long range radio link is done in another way: the low-frequency message signals are modulated upon a high-frequency carrier wave, i.e. a suitable parameter of the carrier wave, e.g. the amplitude, the frequency or the phase is changed according to the message signal. Now, the modulated carrier wave can be transmitted by transmission lines or wireless by the help of antennas; at the receiver side the original signal is obtained by demodulation. There are different kinds of modulations and demodulations which derive from the endeavour to keep parasitic effects as small as possible and in addition, to be able to transmit per message channel as many signals as possible. The wide area of the modulation technique acts like a connection between the information technique and its methods on one hand and the RF technique in the classical meaning on the other hand.

1.2 Types of modulation

If the amplitude, e.g. the voltage amplitude, as well as the phase of the carrier signal is determinined as a time dependent value of the signal, the most common representation of a modulated signal will be given:

\[ u_{RF}(t) = u_s(t) \cos (\phi(t)) = u_s(t) \cos \left( \int_{t_0}^{t} \omega(\tau) d\tau \right). \]  \hspace{1cm} (1)

In the simpler modulation methods only one (in each case) of these parameters is modified according to the signal, in more complicated ones like the quadrature modulation combinations are also used.

The best known types of modulation are presented in the following table – under designation of the relevant parameters:
2 Amplitude modulation (AM)

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The kinds of modulation which are characterised with * are the topics of this laboratory experiment; for a wide overview you may refer the following literature: /1-3/

The laboratory experiment deals mainly with the topic “generation of modulation signals”.

**Important:** Modulation is always connected with a non-linear procedure; in every method that is demonstrated in the following think about wherein the non-linear operation exists.

## 2 Amplitude modulation (AM)

### 2.1 Theory

Principle: The amplitude of a high frequency carrier will be **modulated** according to the “message” (= low-frequency signal). Assumed is an arbitrary signal $s(t)$ that is to be transmitted and it is normalized to have the standardisation

$$|s(t)|_{\text{max}} = 1.$$  

The real valued representations of voltage curves are:

- **unmodulated carrier:**
  $$u_C(t) = \hat{u}_C \cos(\omega_C t)$$  

- **signal voltage:**
  $$u_s(t) = \hat{u}_s s(t)$$  

- **modulated carrier:**
  $$u_{AM}(t) = \hat{u}_{AM}(t) \cos(\omega_C t)$$  
  with  
  $$\hat{u}_{AM}(t) = \hat{u}_C + u_s(t)$$  
  $$= (\hat{u}_C + \hat{u}_s s(t)) \cos(\omega_C t)$$  
  $$= \hat{u}_C (1 + ms(t)) \cos(\omega_C t)$$  

with the **modulation index:**

$$m = \frac{\hat{u}_s}{\hat{u}_C}$$  

Furthermore, the high-frequency carrier wave has an envelope in the shape of the low-frequency signal voltage, see Fig. 1.

As a simplification in the following a perfectly sinusoidal LF-signal is assumed (complicated signals, e.g. rectangular pulses, can be generated from an overlap of those sinusoidal signals in a Fourier series):

$$u_s(t) = \hat{u}_s \cos(\omega_s t + \phi_s).$$

For the theoretical consideration often it is better to exchange to complex calculation with phasor representation which leads to the following expressions:

- **unmodulated carrier:**
  $$u_C(t) = \hat{u}_C e^{j\omega_C t}$$
2 Amplitude modulation (AM)

Fig. 1: Time signal of an AM signal.

signal voltage: \[ u_s(t) = \hat{u}_s e^{j\omega_s t} \] (9)

modulated carrier: \[ u_{AM}(t) = \hat{u}_C e^{j\omega_C t} + \frac{\hat{u}_s}{2} e^{j(\omega_C + \omega_s) t} + \frac{\hat{u}_s}{2} e^{j(\omega_C - \omega_s) t} \] (10)

Interpretation: The real modulated carrier wave \( u_{AM}(t) \) can be demonstrated in the complex plane by six complex vectors which are rotating with time. To make it easier, the conjugate complex vectors are normally left out so that finally a demonstration of three phasors according to Fig. 2 results. The phasor of the carrier wave rotates constantly with the angular frequency \( \omega_C \) around the origin, the phasors which are fixed at the vertex (vector addition) rotate relative to the carrier wave with the angular frequencies \( +\omega_s \) and \( -\omega_s \), respectively.

The bandwidth of an AM signal is:

\[ B = 2f_M, \]

where \( f_M \) is the largest frequency that is contained by the signal.

The AM in the shape it is shown here is known, e.g. from radio broadcast, where it is utilised in

- long-wave range: \( 140 \text{kHz} \ldots 300 \text{kHz} \),
- medium-wave range: \( 525 \text{kHz} \ldots 1600 \text{kHz} \),
- short wave range: \( 1.8 \text{MHz} \ldots 30 \text{MHz} \).

Fig. 2: Vector representation of AM.
Every radio listener knows the effect of noise in these ranges. This effect can be caused by external sources, e.g. ignition sparks in the car.

A further source of error is the transmission path itself. Generally, errors can be caused by attenuation distortion or phase distortion. Linear distortions (e.g. different attenuation of the sideband frequency) and non-linear attenuation (generation of additional unwanted combination frequencies) are differentiated.

2.2 Generation of the amplitude modulation (AM)

2.2.1 Double sideband modulation with the carrier, envelope modulation

In the simplest case the AM signal can be generated with the help of a diode modulator, where non-linear behavior of the diode characteristic is utilized. If the non-linear I-U-characteristic of a diode is charged with an additive superposition of two signals of different angular frequencies $\omega_1, \omega_2$, in the general case an output signal is produced which contains all combination frequencies and their harmonics (this are multiples of the basic oscillation):

$$\omega_{mn} = |m\omega_1 \pm n\omega_2|, \quad m, n = 0, 1, 2 \ldots \quad \text{w/o} \quad m=n=0$$

and maybe a DC part ($m=n=0$). If with the help of the following band pass filter the unwanted spectral parts are suppressed, the AM signal can be gained out of it.

For small rejections the characteristic of a diode in the operating point can be described by a quadratic approximation:

$$i(u) = g_1 u + g_2 u^2,$$

The superposition (addition) carrier- and signal voltage are given on the diode:

$$u(t) = u_C(t) + u_s(t),$$
$$u_C(t) = \tilde{u}_C \cos (\omega_C t),$$
$$u_s(t) = \tilde{u}_s \cos (\omega_s t).$$

In Fig. 3 a diode modulator is demonstrated. With it the measuring tasks shall be carried through.
2.2.2 Double sideband modulation with suppressed carrier (DSB-SC)

In AM all three spectral frequencies will be transmitted, although, the information is just in the two sidebands (more definite: in the distance of the side oscillations to the carrier), but not in the carrier itself. The carrier has the constant frequency and amplitude, therefore, it has no information content, otherwise, especially in the carrier is the largest part of the transmitting power. By suppression of the carrier on the transmitter side this power could be radiated more efficiently as an additional sideband power in order to aim at larger distances. If the carrier is not completely suppressed, carrier recovery in the receiver is possible; this is in practice absolutely necessary, because as a rule, the carrier is required for the demodulation.

According to the circuit the desired output signal—the complete carrier suppression by the multiplication of a carrier- and a baseband signal—can be generated with the aid of, e.g. a ring modulator, technically realised with four diodes arranged in ring-shape, see Fig. 4.

![Fig. 4: Circuit diagram of a ring modulator.](image)

Today this problem can certainly be solved more elegant by an integrated circuit, e.g. with a **product modulator**; this component multiplies two voltage values with each other. In the experiment the balanced mixer IC “SO 42P” of the company Siemens is used.

2.3 Demodulation of AM

In demodulation it is distinguished between **coherent** and **incoherent** demodulation. The coherent demodulation is done by a down-mixing with a carrier frequency equal to the local oscillator frequency, the incoherent demodulation is done in a shape of a signal rectification, e.g. by a diode rectifier.

2.3.1 Envelope detector with a rectifier diode and a low pass filter

By regarding the AM signal in the time domain it can be seen that a demodulation is quite easy to carry through by the signal rectification, the high frequency parts (carrier frequencies) are removed afterwards with the aid of a low pass filter. The impatient radio home constructor receives the LF-signal even without the insertion of a separate low pass in a circuit, because of
the connected loudspeakers which cannot follow the RF-signal which results from its mechanic inertia and therefore acts like a low pass.

The Fig. 5 shows the half-wave rectifier as the simplest example for an envelope modulator.

In order not to falsify the receiver signal, the following two requirements are to lay on the capacitor \( C \):

- \( C \) needs to be as big as it is necessary to approximate a short circuit for the RF-signal:
  \[
  \tau = R_L C \gg \frac{1}{\omega_{RF}}. \tag{16}
  \]

- Furthermore, the time constant \( \tau = R_L C \) needs to be chosen so that the voltage at the storage capacitor is still able to follow the modulation voltage:

The output voltage as function of the envelope voltage due to the \( R-C \) combination is:

\[
  u_{\text{out}}(t) = u_{\text{env}}(t) e^{-\frac{t}{\tau}}. \tag{17}
\]

As of \( \frac{t}{\tau} \ll 1 \) during a period, the exponential function may be approximated using a Taylor series expansion as:

\[
  u_{\text{out}}(t) \approx u_{\text{env}}(t) \left(1 - \frac{t}{\tau}\right). \tag{18}
\]

The envelope voltage have to follow the RF signal fast enough, therefore:

\[
  \left| \frac{du_{\text{out}}(t)}{dt} \right| = \frac{u_{\text{env}}(t)}{\tau} \geq \left| \frac{du_{\text{env}}(t)}{dt} \right| \tag{19}.
\]

The envelope voltage (in the upper plane) as function of the modulation index and baseband frequency is:

\[
  u_{\text{env}}(t) = A \left(1 + m \cos(\omega_{LF} t)\right), \tag{20}
\]

\[
  \Rightarrow \quad \frac{du_{\text{env}}(t)}{dt} = -A m \omega_{LF} \sin(\omega_{LF} t), \tag{21}
\]

\[
  \frac{A \left(1 + m \cos(\omega_{LF} t)\right)}{\tau} \geq A m \omega_{LF} \sin(\omega_{LF} t) \quad \forall t, \tag{22}
\]

\[
  \Leftrightarrow \quad \tau \leq \frac{1 + m \cos(\omega_{LF} t)}{m \omega_{LF} \sin(\omega_{LF} t)} \quad \forall t. \tag{23}
\]
The first derivation of the right hand side has to be zero (and the second derivation bigger than zero) to get the minimal value. Using the quotient rule it follows:

\[
\frac{d}{dt}\left(\frac{1 + m\cos(\omega_{LF}t)}{m\omega_{LF}\sin(\omega_{LF}t)}\right) = 0,
\]

(24)

\[-\left(m\omega_{LF}\sin(\omega_{LF}t)\right)^2 - \left(1 + m\cos(\omega_{LF}t)\right)\frac{m\omega_{LF}^2\cos(\omega_{LF}t)}{(m\omega_{LF}\sin(\omega_{LF}t))^2} = 0,
\]

(25)

\[\Rightarrow m\sin^2(\omega_{LF}t) + m\cos^2(\omega_{LF}t) + \cos(\omega_{LF}t) = 0,
\]

(26)

\[\Rightarrow \cos(\omega_{LF}t) = -m,
\]

(27)

(26) : \(\sin(\omega_{LF}t) = \sqrt{1 - m^2},\)

(28)

(23) : \(\tau \leq \frac{1 - m^2}{m\omega_{LF}\sqrt{1 - m^2}} = \frac{\sqrt{1 - m^2}}{m\omega_{LF}}.\)

(29)

Altogether this is the condition:

\[
\frac{1}{\omega_{RF}} \ll R_L C \leq \frac{\sqrt{1 - m^2}}{\omega_{LF} m},
\]

(30)

That means for the maximum possible modulation index \(m=1\) no distortion-free demodulation is possible.

### 2.3.2 Product demodulation (coherent demodulation)

The amplitude-modulated signal will be multiplied with a reference signal whose frequency is the same as the carrier frequency. The first case is the

**Double-sideband AM signal with carrier**

\[
u_{AM}(t) = \hat{u}_C \left(1 + m\cos(\omega_{LF}t)\right)\cos(\omega_C t),
\]

(31)

\[
u_{Ref} = \hat{u}_{Ref}\cos(\omega_C t + \phi_C),
\]

(32)

\[u_{Dem}(t) = k u_{AM}(t) u_{Ref}(t),
\]

(33)

\[= k \hat{u}_C \hat{u}_{Ref} \frac{1}{2} \left[1 + m\cos(\omega_{LF}t)\right] \left(\cos\phi_C + \cos(2\omega_C t + \phi_C)\right),
\]

(34)

\(\phi_C\) : phase difference for the carrier signal, \(k\) : demodulator constant.

Result: The phase difference between the carrier and the reference signal, which is the **phase error**, results in an attenuation of the DC component and of the LF-amplitude, but no further distortion. Remedy can be given by a carrier regain, e.g. according to Fig. 6.

The demodulator is applied in the last stage of the receiver (after the last down-conversion of the input signal). The limiting amplifier acts as an comparator, i.e. it has a very high gain.
and is works in saturation. The following bandpass filter consists mostly of an $L$-$C$ resonance circuit only having the bandwidth $B$ which is smaller than the minimal LF frequency, i.e. $B<f_{\text{LF, min}}$.

The second case is the

**AM signal without carrier**

\[ u_{\text{AM}}(t) = \frac{1}{2} \hat{u}_{\text{AM}} \left[ \cos((\omega_C + \omega_{\text{LF}})t) + \cos((\omega_C - \omega_{\text{LF}})t) \right]. \tag{35} \]

reference signal: \( u_{\text{Ref}} = \hat{u}_{\text{Ref}} \cos(\omega_C t + \phi_C) \), \tag{36}

\[ u_{\text{Dem}}(t) = u_{\text{AM}}(t) u_{\text{Ref}}(t) \tag{37} \]

\[ \ldots \]

\[ u_{\text{Dem}}(t) = \frac{1}{2} \hat{u}_{\text{AM}} \hat{u}_{\text{Ref}} \left[ \cos \phi_C + \cos(2\omega_C t + \phi_C) \right] \cos(\omega_{\text{LF}} t). \tag{38} \]

relevant component after filtering:

\[ u_{\text{LF}}(t) = \frac{1}{2} \hat{u}_{\text{AM}} \hat{u}_{\text{Ref}} \cos(\omega_{\text{LF}} t) \cos(\phi_C). \tag{39} \]

Result: the phase error leads to an attenuation of the LF-component, there is no distortion shown. In practice the problem can also be avoided by a carrier regain, for this there are diverse special circuits. But here, they are not discussed in more details.

### 3 Frequency modulation (FM)

#### 3.1 Theory

Principle: The frequency of a high frequency carrier will be modified according to the signal that is to be transmitted:

unmodulated carrier: \( u_C(t) = \hat{u}_C \cos(\omega_C t + \phi_C) \tag{40} \)

signal voltage: \( u_s(t) = \hat{u}_s s(t) \tag{41} \)
Instantaneous angular frequency of the modulated carrier:

\[ \omega(t) = \omega_C + k_f u_s(t) = \omega_C + \Delta \omega(t) \]  
(42)

\( k_f \): constant of the modulator

General rule:

\[ \omega(t) = \frac{d\phi(t)}{dt} \]  
(43)

i.e. the phase of the modulated carrier is given by the following:

\[ \phi(t) = \omega_C t + k_f \int_0^t u_s(\tau) d\tau + \phi_C \]  
(44)

and therewith:

\[ u_{FM}(t) = \hat{u}_C \cos \left( \omega_C t + k_f \int_0^t u_s(\tau) d\tau + \phi_C \right) . \]  
(45)

Simplifying assumption: the message signal is a harmonic signal:

\[ u_s(t) = \hat{u}_s \cos(\omega_s t) , \]  
(46)

\[ \int_0^t u_s(\tau) d\tau = \hat{u}_s \frac{1}{\omega_s} \sin(\omega_s t) , \]  
(47)

\[ \Delta \omega(t) = k_f \hat{u}_s \cos(\omega_s t) = \Delta \hat{\omega} \cos(\omega_s t) , \]  
(48)

\[ u_{FM}(t) = \hat{u}_C \cos \left( \omega_C t + \Delta \hat{\omega} \frac{\sin(\omega_s t)}{\omega_s} + \phi_C \right) , \]  
(49)

**frequency deviation:** \( \Delta \hat{\omega} \),

**modulation index:** \( \eta = \frac{\Delta \hat{\omega}}{\omega_s} \).  
(50)

Fig. 7 shows typical characteristics of an FM signal.

\[ \frac{u_{FM}(t)}{\hat{u}_C} \]

Fig. 7: Time characteristics of an FM signal.

Junction to the complex syntax:

\[ u_{FM}(t) = \text{Re} \left\{ \hat{u}_C e^{j(\omega_C t + \phi_C)} e^{j\eta \sin(\omega_s t)} \right\} . \]  
(52)
The second exponential term can be demonstrated with the help of the Fourier analysis as:

\[ e^{j\eta \sin(x)} = \sum_{n=-\infty}^{+\infty} J_n(\eta) e^{jn\omega_s} \],

(53)

\[ J_n: \text{Bessel function of 1st kind and of } n\text{-th order, it is applied:} \]

\[ J_{-n}(\eta) = (-1)^n J_n(\eta), \quad n = 1, 2, 3 \ldots \]

(54)

Therewith the frequency modulated voltage results to:

\[ u_{FM}(t) = \text{Re} \left\{ \hat{u}_C \sum_{n=-\infty}^{+\infty} J_n(\eta) e^{j[\omega_C+n\omega_s]t+\phi_C} \right\}. \]

(55)

With help of (55) the spectrum of an FM signal can now also be determined, hereafter, \( u_{FM}(t) \) results from the sum of an infinitely large number of frequencies

\[ \text{with the frequencies: } \omega_n = \omega_C \pm n\omega_s \]

(56)

\[ \text{and the amplitudes: } \hat{u}_n = \hat{u}_C J_n(\eta). \]

(57)

Thus, an infinitely large bandwidth \( B \) theoretically results, in practice you normally find a bandwidth of

\[ B = 2(\eta + 1)B_s \quad \text{Carson-bandwidth} \]

(58)

as sufficient, because the signal power basically sticks in the frequencies which are close to \( \omega_C \). \( B_s \) is the bandwidth of the baseband signal.

Example: By the so-called single-tone frequency-modulation it is applied by a very small modulation index

\[ \eta \ll 1: J_0(\eta) \approx 1, \quad J_1(\eta) \approx \frac{\eta}{2}. \]

In Fig. 8 the facts are demonstrated.

![Fig. 8: Phasor diagram and amplitudes spectrum of the single tone-frequency modulation.](image)

**Fig. 8**: Phasor diagram and amplitudes spectrum of the single tone-frequency modulation.
3.2 Generation of frequency modulated signals

Frequency modulated signals are most easily generated with the help of a voltage controlled oscillator (VCO: Voltage Controlled Oscillator), whereas the frequency of the carrier oscillator is directly controlled by the signal voltage. The voltage dependency of the oscillator frequency results from, e.g., a controlled capacitor (varactor diode) in the frequency-determining resonant circuit of the oscillator.

4 Phase modulation (PM)

Principle: The phase of a high frequency carrier will be modulated according to the signal which is to be transmitted. The most important variables are:

\[
\begin{align*}
\text{unmodulated carrier:} & \quad u_{PM}(t) = \hat{u}_{PM} \cos(\phi(t)) \\
\phi(t) & = \omega_C t + k_p u_s(t) + \phi_0 \\
\omega(t) & = \frac{d\phi(t)}{dt} = \omega_C + k_p \frac{du_s(t)}{dt} \\
\text{signal voltage:} & \quad u_s(t) = \hat{u}_s \cos(\omega_s t) \\
\text{PM modulated signal:} & \quad u_{PM}(t) = \hat{u}_{PM} \cos(\omega_C t + \Delta \hat{\Phi} \cos(\omega_s t) + \phi_0) \\
\text{phase deviation:} & \quad \Delta \hat{\Phi} = k_p \hat{u}_s
\end{align*}
\]

Frequency and phase modulation are more commonly referred to as angular modulation. It is to realize that the PM signal can be generated with the help of an available FM-modulator if the LF-signal was differentiated before, see Fig. 9.

\[u_2(t) = -RC \frac{du_1(t)}{dt}\]

Fig. 9: PM-modulator.
Demodulation of FM signal

Generally, FM-demodulators are known as *discriminators*. In a large number of discriminators (for example slope, balanced, phase, relation discriminators) the FM signal is first converted into an AM signal and then it is given to an AM-demodulator (rectifier). In most cases, before the discriminator a strong amplitude limitation is done in order to disable troubles by high frequency amplitude oscillations, see Fig. 10. This process has no damaging effects in the LF-wanted signal, because the information is just contained in the sequence of zeros (frequency) of the signal.

![Fig. 10: FM demodulator.](image)

In the simplest case, the conversion of FM into AM can take place with the help of an inductor by a *L-discriminator*, see Fig. 11. The frequency dependent value of the impedance leads to (for constant current) a frequency dependent voltage decrease at the inductor and thereby to the wanted AM signal.

![Fig. 11: L-discriminator.](image)

In theory, this simple discriminator has an optimal converter characteristics because of the linear dependency between the frequency and the voltage (for constant current), in practice, the inductivities with an increasing frequency lose very fast their ideal characteristics so that the L-discriminator is just restrictively suitable for the demodulation of the FM signals: because of the fringing capacitances by coils the inductor value needs to be chosen very small, this leads just to small voltage amplitudes.

![Fig. 12: Equivalent circuit diagram of the slope discriminator.](image)

In the *slope discriminator* the FM signal will be given to a *L-C* resonator circuit whose resonance is a bit shifted to the carrier frequency, see Fig. 12 and 13. By choosing the operating point it should be considered that the operating point itself should be in the linear range of
the transfer characteristic, therefore in the bandwidthpoint of the resonator circuit, i.e. at 3 dB attenuation. The changing frequency involves again a changing amplitude. Another improvement, especially concerning the linearity, is received by the balanced discriminator, see Fig. 14.

In principle, this one consists of two symmetrically arranged slope discriminators whose resonance curves are shifted symmetrically \((f_u, f_o)\) and whose output voltages get subtracted from each other whereby altogether the converter characteristic results, which is given in Fig. 15. The balanced circuit effects that the non-linearities of the bended slopes of both single circuits cancel out each other partially; moreover, the modulation range will be raised compared to the simple slope discriminator.

For the use of monolithic realisations other methods are more suitable, e.g. a quadrature modulator, a so-called coincidence demodulator, which is built up with the help of the component “SO 41” from Siemens. In the following, the mode of operation of this quadrature modulator shall be briefly shown. The Fig. 16 shows the necessary components in the block diagram. The phase shifter is designed so that at the carrier frequency the phase shift between \(u_1\) and \(u_2\) is just about 90°, and that near by this frequency it changes linearly with the frequency deviation, see Fig. 17. Both signals are given to a phase comparator in the shape of an analog-multiplier; after the low pass filtering the LF-signal results finally.
Fig. 15: Voltages at the balanced discriminator.

Fig. 16: Coincidence demodulator.

**Literature**


At a glance:

Fig. 17: Voltages at the coincidence demodulator.
6 Spectrum Analysers

In the following test a spectrum analyser measures the performance of the respective frequency parts of the oscillator. To be able to judge the influence of this measuring instrument on the results, its basic construction must be known.

A simple spectrum analyser can be realised with a direct receiver (Fig. 18).

![Fig. 18: Spectrum Analyser (straight receiver).](image)

After passing through an electrically tunable filter, the input signal reaches an equality check and then passes on to the $y$-direction of a $x$-$y$ display. The $x$-direction of the display is controlled by a saw-tooth generator, which concurrently provides the control-signal for the tunable filter. Thus, each point of the $x$-axis of the display corresponds to one centre frequency of the filter and consequently to one spectral component of the input signal, the amplitude of which is shown on the $y$-axis.

A disadvantage of this circuit is that it requires a broad-band signal-processing in RF range, which requires broad-band amplifiers and increases the impact of noise on the measurement. Moreover, a filter is required to be tunable over a broad range and with stable bandwidth, which is very difficult in practise. Therefore straight-receiving spectrum analysers are only used in LF ranges and the tunable filter is replaced by a series of many filters with constant centre frequency. They are switched one by one in the signal path.

A spectrum analyser more suitable for the RF range is based on the super-heterodyne principle (super-heterodyne receiver, Fig. 19).

In this case the tunable filter is replaced by a IF-filter with fixed centre frequency, a mixer and a VCO. The input signal is prefiltered (to suppress image frequency) then mixed with the VCO signal, fed to an IF-filter and an amplifier and after being rectified drives to the $y$-direction of the display. The sawtooth generator controls the frequency of the VCO and the $x$-direction of the display. Each VCO frequency corresponds to an input frequency, which is, after mixing with the oscillator signal, the intermediate frequency and therefore can pass the IF-filter. As a result each point of the $x$-axis of the display corresponds again to one frequency component of the input signal. In contrast to the direct receiver only a narrow-band receiver with quite
low frequencies (IF-range) and frequency-fixed filter is required which can be realised at a high quality. Due to the disadvantages of a direct receiver in the RF area only spectrum analysers based on superheterodyne principle are used, in the RF range.

While measuring with spectrum analysers please note that the filter takes some time to resonate. The smaller the bandwidth of the filter is, the more the time it takes. If the Sweep-time, in which the filter or the oscillator are tuned, is chosen too short, the filter cannot resonate and the spectrum displayed is distorted. With modern spectrum analysers the Sweep-time is automatically adjusted to the frequency interval to be measured and to the bandwidth of the filter.

Further components of a spectrum analyser (which are not shown in the block diagram) are the input attenuator to adjust the input signal to the following levels, a low-pass filter (videofilter) between detector and display to smooth the measured curve and also levels to control the measuring and to facilitate the handling. In modern devices the rectified signal is fed to an A/D-converter and digitally stored. Thus, it can be displayed and processed independant of the sawtooth generator.

**Literature**


7 Questions and Problems on the Lab

7.1 General Questions

**Problem 1:** Which voltage corresponds to 1 dBµ (=dB for µV)? and which power in dBm (=dB for mW) corresponds to this at 50 Ω?

7.2 About amplitude modulation

**Problem 2:** Draw the AM time domain signal for a modulation index \( m \) larger than 1 (>100 %).

**Problem 3:** Derive from (6) the frequency spectrum of an AM signal for the special case of the sinusoidal signal \( s(t) = \cos(\omega_s t) \).

Determine the powers that are transmitted by the appearing frequencies.

Which fraction of the transmitted total power is in the separate partial oscillation, i.e. the carrier and the two sidebands, for the special case \( m = 1 \)?

**Problem 4:** Draw the time signal which is obtained shown by the addition of two sinusoidal signals of almost the same angular frequency \( \omega_C \approx \omega_s \) but very different amplitudes.

In the case the amplitude have the same value, the superposition is called **beat signal**. How now the signal looks like?

Compare the amplitude spectrum of a beat signal with that of an AM signal (drawing).

**Problem 5:** Why do AM signals react more sensitively to ignition sparks in comparison to FM signals?

**Problem 6:** Draw an arbitrary spectrum of three spectrum components. Now, sketch for the cases of linear and non-linear distortions the modified spectrum which is in contrast to the error-free transmission.

**Problem 7:** Which spectrum results from inserting the superposition of two voltages (13) into the nonlinear characteristic (12)? Sketch the spectrum.

Which constraints are to be put on the band pass which is following the diode to achieve the double sideband modulation signal? Insert the band pass characteristic into the drawing of the spectrum.

**Problem 8:** Demonstrate that by the multiplication of a carrier- and an LF-signal an AM signal without a carrier is generated. Assume for both signals harmonic oscillations.

**Problem 9:** Think about how the circuit in Fig. 4 works and sketch the time functions of the voltages; the diodes can be replaced by switches in the first approximation. These switches can be switched on and off by a positive or negative half cycle of the carrier signal \( u_C(t) \).

Why is the generation of an AM signal without carrier possible using a rectangular signal instead of a sinusoidal signal, cf. the previous question?
Problem 10: Think about the mode of function of the circuit in Fig. 5; for this you are allowed to regard the diode as an ideal circuit. Sketch the time domain output signal without and with the $R$-$C$ combination.

Problem 11: Calculate the capacitance $C$ for the available demodulator circuit in Fig. 5 for the values:

$$f_{RF}=10.7 \text{MHz}, \quad f_{LF}=20 \text{kHz}, \quad R_L=60 \text{k}\Omega, \quad m=0.99.$$  

Problem 12: Give a drawing of the calculated spectrum at point M and at the output LF of the block diagram in Fig. 6 in the case the input signal is a DSB-AM with carrier. Insert a reasonable filter characteristic into this spectrum.

7.3 About frequency modulation

Problem 13: For which value of $\eta$ does the carrier disappear in an FM signal?

Problem 14: Sketch the time signal of the FM signal, which is shown in Fig. 7, page 9, after the conversion into an AM signal.

Which fundamental difference exists between this signal and a “usual” AM signal with the corresponding envelope?

Problem 15: Why does the maximum frequency deviation at a slope discriminator need to be considerably smaller than half of the bandwidth of the resonator circuit?

7.4 About spectrum analyzer

Problem 16: Fig. 20 shows the simplified block diagram of a spectrum analyzer. The letters A to J indicate adjustable parameters of the analyzer. The names of the corresponding operational controls are listed below the block diagram.

Assign the letters A to J to the corresponding operational controls.

7.5 Signal analysis

Problem 17: Calculate the Fourier transform of a signal at 50 Hz with a sampling rate of 250 Hz. Use Matlab, Octave, Scilab, or a similar program for your simulation.

At which frequency is the maximum? Bring along the printed version of the time and frequency domain results to the lab date.
Fig. 20: Block diagram of a spectrum analyzer.

Gain Adjust
Scale dB/div
Sweep Time
Reference Level
Input Attenuation
lin ↔ dB
RBW = Resolution Bandwidth
Video Bandwidth
Center Frequency
Span
8 Measuring Tasks

Stand: 21st June 2021

Double sideband modulation with carrier (AM)

Modulator: Diode modulator DIODE MOD in the rack AMPLITUDE MODULATOR
inputs: LO IN (RF-signal), NF IN
outputs: LO OUT und NFOUT (signal of sums, measurement with the oscilloscope, not needed),
RF OUT (AM signal, measurement with the spectrum analyzer or the oscilloscope)
RF source: HP 8648B, unmodulated, frequency 10.7MHz, level $-60$ dBm
LF source: HM8030, 10kHz-sinusoidal signal, signal voltages about $0.6\, V_{pp}$

Task 1: Consider the AM signal with the help of the oscilloscope in the time domain and make a drawing of the figure. Determine the modulation index.

Hints: Which trigger signal should be used? Why is it important to adjust the amplitude of the LF signal with all connections necessary for the measurement?

Task 2: Modulation trapeze: Activate the XY-mode of the oscilloscope (Mode button: 6 squares). Consider how the signals are displayed, and determine again the modulation index.

What advantages are provided by this method in comparison to the Task 1?

Task 3: Consider the modulated signal with the help of a spectrum analyzer in the frequency range. Determine the modulation index in so doing.

Task 4: The carrier will now be modulated with a rectangular signal. Consider how the spectrum of the AM signal looks like. Sketch the signal as seen in the spectrum analyzer.

Double sideband modulation with suppressed carrier (DSB)

Modulator: balanced mixer BAL MIXER in the rack AMPLITUDE MODULATOR
inputs: LO IN (RF-signal), NF IN
output: RF OUT
RF source: as above, but level $-15$ dBm
LF source: as above but again a sine signal

Task 5: Consider the modulated signal with the help of the spectrum analyzer in the frequency range. How large is the (theoretical infinite big) carrier suppression, i.e. the difference value between the carrier an the sideband in dB, actually?

Task 6: Consider the signal with the help of the oscilloscope in the time range and make a drawing.
Envelope rectifier with a detector diode and low pass

demodulator: DIODE DEMOD in the rack AMPLITUDE DEMODULATOR
With the help of the rotary switch three different values can be
adjusted with the capacity C.
AM signal: HP 8648B, modulation AM, carrier frequency 10.7 MHz, output
level $-15\,\text{dBm}$, modulation index 50%, EXT AC, MOD ON.

Connect the output of the function generator (adjustment 10kHz-sinusoidal signal) with the
port MOD INPUT of the AM/FM signal generator. The signal level at the function generator
needs to be adjusted with help of the oscilloscope in a way, that the amplitude at the signal
generator is 1 V. The load load of the modulation input of the signal generator has to be
considered.

**Task 7:** Consider the demodulated signal for different values of $C$ with the help of the oscil-
oscope.

What happens if $C$ is chosen respectively too big or too small? Zoom eventually the
measured curve. Make a drawing.

Product demodulation (coherent demodulation)

D demodulator: product demodulator SYNCHRO DEMO in the rack AMPLI-
TITUDE DEMODULATOR. The port NFOUT conforms with the
point M in the illustration 6.
AM signal: as above, but increase the frequency of the modulation signal
to 20kHz.

**Task 8:** Make a qualitative drawing of the spectrum at the point M of Fig. 6, page 8 (output
of the multiplier).

Check your results of the appropriate lab’s preparation question by an adequate mea-
surement with the spectrum analyzer.

Generation of frequency modulated signals

FM signal: HP 8648B, modulation FM, carrier frequency 10.7 MHz,
output level $-15\,\text{dBm}$, EXT AC, MOD ON

**Task 9:** Without VCO! 20kHz FM-Modulation (=modulation signal is 20kHz). The signal
level at the function generator needs to be adjusted with help of the oscilloscope in a
way, that the amplitude at the signal generator is 1 V. The load of the modulation
input of the signal generator has to be considered.

Determine $\eta$ for the minimal carrier as precise as possible first by adjusting the frequency
deviation $\Delta \omega$, finally with the signal angular frequency $\omega_s$.

For which value of $\eta$ does the carrier of the FM signal disappear? Compare the value
with the calculated value of the appropriate preparation question.

Hint: $\Delta f$ can be adjusted on the HP 8648B with the arrow keys between FUNCTION
and DATA block. Alternatively it can be entered directly via the number pad.

**On-Off-Keying**

**Signal source**  
SMA 100 Pulse Generator  
Pulse Mode Double, Pulse Period 10 µs, Pulse Width 2 µs,  
Double Pulse Width 1 µs, Double Pulse Delay 3 µs,  
Pulse Modulator (State On, Source Pulse Generator),  
RF (1 GHz, Level 0 dBm)

**Oscilloscope:**  
RTO 1044  
Channel 1 RF out:  
signal with DC block, input Ch1: 50 Ω coupling, bandwidth full  
Channel 2 pulse signal for modulation:  
pulse video with 10 dB attenuation, input Ch2, 1 MΩ, bandwidth full  
Channel 3 pulse sync with 10 dB attenuation:  
input Ch3, 1 MΩ, bandwidth full  
Horizontal divs: 10 ns for trigger.  
vertical divs: 1 V.  
trigger: channel 3, edge, turn up the trigger level.  
horizontal divs: 2 µs.

**Task 10:** Analyse the signal generated with the source SM100 with help of the oscilloscope. Choose an appropriate trigger setting.

**Task 11:** Save the logged signal and analyse it with Matlab. Display the signal correctly in the time domain and verify the selected frequency using the spectrum.

**Task 12:** Which bit sequence should the signal represent? An available signal represents a 1 and no amplitude stand for a 0.
8 Measuring Tasks