Please note the **Important Hints** on a separate sheet.

## 1 Basics

In this lab S-parameter measurements in the frequency range 100 kHz–6 GHz using the coaxial transmission lines will be performed.

There are different transmission line types to guide radio frequency (RF) signals while the coaxial line technique is the most common one. The frequency range starts at DC and reaches up to some GHz. In the coaxial measurement techniques the upper limit is generally about 40 GHz, but there are (very lossy) coaxial cables available for frequencies up to 110 GHz, too.

### 1.1 Scattering Matrix of a Linear Twoport

In RF engineering the description of N-ports using parameters based on voltages and currents is not very reasonable since voltages and currents are no unambiguous values (e.g. rectangular waveguide, higher (waveguide) modes in a coaxial line).

The electrical description of N-ports using scattering parameters is much more suited since it is based on incident and reflected power waves.

![Linear twoport including reference planes.](image)

Figure 1 shows a linear twoport; both ports are normalized to $Z_0$. The relation between the parameters is:

\[
b_1 = s_{11}a_1 + s_{12}a_2 ,
\]

\[
b_2 = s_{21}a_1 + s_{22}a_2 ,
\]

or in matrix notation

\[
\begin{pmatrix}
b_1 \\
b_2
\end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22}\end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}
\]

\[
\vec{S} : \text{complex scattering matrix,}
\]

\[
a_n : \text{time independant complex wave amplitude, incident to port } n ,
\]

\[
b_n : \text{time independant complex wave amplitude, reflected at port } n .
\]

The interconnection is only possible if both ports have the **same** characteristic impedance, the **same** cross section, the **same** geometrical orientation, and the **same** mode is considered.
Otherwise, the interconnect is a discontinuity which may be described by another scattering matrix.

The parameters \( s_{ik} \) are defined in the following way (or may be measured in this kind):

\[
\begin{align*}
\xi_{11} &= \left. \frac{b_1}{a_1} \right|_{a_2=0} = \text{Complex input reflection coefficient of port 1, if port 2 is matched.} \\
\xi_{12} &= \left. \frac{b_1}{a_2} \right|_{a_1=0} = \text{Complex transmission coefficient from port 2 to port 1, if port 1 is matched.} \\
\xi_{21} &= \left. \frac{b_2}{a_1} \right|_{a_2=0} = \text{Complex transmission coefficient from port 1 to port 2, if port 2 is matched.} \\
\xi_{22} &= \left. \frac{b_2}{a_2} \right|_{a_1=0} = \text{Complex input reflection coefficient of port 2, if port 1 is matched.}
\end{align*}
\]

1.2 Properties of N-ports

Reciprocity In the case the scattering matrix is symmetric with respect to the main diagonal, an N-port is reciprocal, i.e. \( s_{ik} = s_{ki} \), \( i \neq k \). This is in either case true for N-ports consisting of passive, linear, and isotropic elements. In matrix notation:

\[
\vec{\xi} = \vec{\xi}^T.
\]  

(Reflection) symmetry An N-port is symmetric with respect to the reflection, if it is reciprocal and additionally, the following relation is valid:

\[
\xi_{ii} = \xi_{kk}.
\]

Lossless An N-port is lossless, if

\[
\sum_{m=1}^{N} \xi_{mk}^* \xi_{ml} = \delta_{kl} = \begin{cases} 
0 & \text{if } k \neq l \\
1 & \text{if } k = l 
\end{cases}
\]

is valid. \( \delta_{kl} \) is called Kronecker delta. The condition in matrix notation

\[
\vec{\xi}^* \vec{\xi}^T = \vec{I}.
\]

This is the so-called unitary condition with \( \vec{I} \) being the unity matrix.

1.3 Measuring Set-up

To determine amplitude and phase of the scattering parameters, a vector measurement set-up is necessary. Complex quantities have to be known, e.g. for the design of matching networks for active devices.
On the other hand a scalar characterization of DUTs (devices under test) is sufficient very often. Scalar means that only the magnitude of the scattering parameters is measured. Figure 2 shows the principle of such a set-up.

The in frequency and power adjustable RF wave delivered from the RF source arrives at the coupler. After the main path of the coupler the DUT is inserted. The power passing the DUT reaches the upper detector diode. This part of the power which is reflected by the DUT is led by the coupler to the lower detector diode.

The output signals of both diodes are displayed after a logarithmic amplification on a plotter. The signal for the $x$-axis comes from the sawtooth generator of the RF source (sweep generator) and is the linear scale of the frequency. Figure 2 shows an oscilloscope instead of a plotter; the setup of this experiment makes use of a PC to display the curves.

### 1.3.1 Detector Diode

By using detector diodes only the incident power of RF signals can be measured, a phase measurement is not possible. Figure 3 shows the characteristic curve of a typical detector.

The output signal of detector diodes is a DC voltage which is at the working point proportional to the incident power. The input voltages of the working area have to be between roughly 350 µV and 25 mV. The working point (WP) is set by using a bias circuit. Easy to handle are so-called zero-bias diodes which means there is no biasing needed. To be more precise, since the diode produces a DC part, which shifts the WP a bit from the zero point, the diode is self-biased.

In Fig. 4 the equivalent circuit of a detector diode is shown. Additionally, an impedance matching circuit and a capacitor working as a low pass filter for the output signal (video signal) is needed. The most common detector diodes are point-contact diodes (crystal diodes),
1 Basics

Fig. 3: Typical characteristic curve of a detector diode.

Schottky diodes (metal semiconductor transition) and back diodes (highly doped p and n areas, Tunnel Effect).

1.3.2 Directional Coupler

A directional coupler is inserted between the RF source and the DUT. Such couplers are characterized generally by the parameters of coupling (e.g. 10 dB, 16 dB, 20 dB), directivity and standing wave ratio (SWR).

In Fig. 5 a wired backward coupler used for the measurement of the reflection coefficient of an unknown device (DUT) is shown. If port 1 is fed by a signal with the power $P_1$, the values $P_2$, $P_3$, and $P_4$, resp. are the powers which drop out of the appropriate ports.

Please pay attention to the fact that the numbering of the four ports is done arbitrarily. Additionally, you have to distinct between backward and forward couplers.

In the measuring set-up the coupler is connected like it is shown in Fig. 5: RF source at port 1, DUT at port 2, load at port 3, and detector at port 4. The power reflected from the DUT should be measured. For the coupler it is:

$$b_2 = s_{21}a_1 + s_{22}a_2 + s_{23}a_3 + s_{24}a_4,$$  \hspace{1cm} (8)

$$b_4 = s_{41}a_1 + s_{42}a_2 + s_{43}a_3 + s_{44}a_4.$$  \hspace{1cm} (9)
Assume an ideal detector and an ideal load:

\[ a_3 = a_4 = 0. \]  \hspace{1cm} (10)

With

\[ a_2 = r b_2, \]  \hspace{1cm} (11)

and \( r = \frac{a_2}{b_2} \) is the reflection coefficient of the DUT:

\[ b_2 = \frac{s_{21}}{1 - r s_{22}} a_1, \]  \hspace{1cm} (12)

\[ \frac{b_1}{a_1} = s_{41} + s_{42} \frac{r s_{21}}{1 - r s_{22}}. \]  \hspace{1cm} (13)

An ideal coupler having a SWR=1 or \( s_{22} = 0 \) and an infinite decoupling attenuation, \( s_{41} = 0 \). It follows in this case:

\[ \frac{b_1}{a_1} = s_{42} s_{21} r. \]  \hspace{1cm} (14)

Due to the simplifications made above the importance of the load, directivity, and SWR is obvious. In the case of poor decoupling and directivity values the detector gets additional power from the source directly which leads to wrong results. This is true also for the matching of the ports (SWR); in this case there are multiple reflections which will be measured from the detector. Directivity is a useful extent for the maximal measurable reflection attenuation of the DUT.

### 1.3.3 Logarithmic Amplifier

The logarithmic amplification of the detector signal having the function

\[ U_{\text{Osci}} = 20 \log \frac{U_{\text{Detekt}}}{U_{\text{Ref}}} \text{ dB} \]  \hspace{1cm} (15)

leads to the normalized diagram in decibel (dB).
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1.4 Calibration

1.4.1 Reflection calibration

For calibration purpose of \( |s_{11}| \) port 2 of the directional coupler is connected with a short circuit, and it will be measured:

\[
\begin{align*}
\tilde{b}_4 / \tilde{a}_1 &= -s_{42}s_{21}. \\
\end{align*}
\]  

(16)

The normalized measured value is

\[
\begin{align*}
\frac{b_4}{b_4} &= \frac{s_{42}s_{21}}{-s_{42}s_{21}} = -r. \\
\end{align*}
\]  

(17)

On measuring the DUT all ports have to be terminated by (ideal) loads. Then we get with:

\[
\begin{align*}
r &= Z_{\text{DUT}}^{\text{L}}, \\
|s_{11}|_{\text{DUT}}^{\text{dB}} &= 20 \log \left| \frac{b_4}{\tilde{b}_4} \right|_{\text{dB}} = 20 \log \left| \frac{b_4}{\tilde{b}_4} \right|_{\text{dB}} - 20 \log \left| \frac{b_4}{\tilde{b}_4} \right|_{\text{dB}}. \\
\end{align*}
\]  

(18)

The subtraction of the calibration values in dB from the measured ones leads directly to the desired calibrated values.

1.4.2 Transmission calibration

The calibration of the detector for \( |s_{21}| \) will be performed in an analogue manner.

The available amplitude at the detector is:

\[
\tilde{b}_2 = s_{21} a_1, \\
\]  

(20)

the measured amplitude after the DUT:

\[
\frac{b_2}{b_2} = s_{21}^{\text{DUT}} a_1. \\
\]  

(21)

We get:

\[
|s_{21}|_{\text{DUT}}^{\text{dB}} = \left| \frac{b_2}{\tilde{b}_2} \right|_{\text{dB}} = \left| \frac{b_2}{\tilde{b}_2} \right|_{\text{dB}} - \left| \frac{b_2}{\tilde{b}_2} \right|_{\text{dB}}. \\
\]  

(22)

The power of the RF source starting from the maximal available power will be reduced by using either adjustable attenuators or directly by adjusting the power in the generator by, e.g. 0 dB, 3 dB, 6 dB, 10 dB. For these input powers the output power will be measured and plotted as function of frequency. Thereafter the real measuring values of the DUT will be plotted into this new “coordinate system”.

Lab course RF Engineering 4: Scalar S-Parameter Meas. (coaxial)
2 Lumped (Concentrated) Elements

All lumped elements have parasitic effects like

- self-inductances and feeding line inductances,
- self-capacitances and housing capacitances,
- frequency dependence of the resistance (skin-effect, proximity effect),
- frequency dependence of the dielectric constant.

If the geometrical dimensions of a lumped element hit the order of magnitude of the wavelength, this element has to be considered as a distributed element.

3 $\frac{\lambda}{4}$ Transformer as Inverter

Inverters are twoports, which are able to transform impedances. Assume at port 2 of an inverter the load impedance

$$z_2 = \frac{Z_2}{Z_0}$$

(23)

the input impedance at port 1 becomes then

$$z_1 = \frac{Z_1}{Z_0} = \frac{k^2}{z_2}, \quad Z_1 = \frac{K^2}{Z_2}$$

(24)

wheras $K$ is the inverter constant and $k$ the normalized inverter constant, resp. and $Z_0$ is the characteristic impedance.

4 Filters Made Up of Short Transmission Lines

4.1 Short Transmission Lines

Fig. 6: Homogenous transmission line of length $l$ whose characteristic impedance $Z_0$ differs from those of the adjacent lines $Z_1$.

The scattering matrix of an ideal transmission line having line length $l$ whose characteristic impedance $Z_0$ differs from the value of the adjoining transmission lines $Z_1$ (see Fig. 6), is

$$S = \frac{1}{1 - r^2 e^{-2\gamma l}} \begin{pmatrix} r(1 - e^{-2\gamma l}) & (1 - r^2)e^{-2\gamma l} \\ (1 - r^2)e^{-2\gamma l} & r(1 - e^{-2\gamma l}) \end{pmatrix},$$

(25)
4 Filters Made Up of Short Transmission Lines

\[ r = \frac{z - 1}{z + 1}, \quad z = \frac{Z_0}{Z_1}. \] (26)

In the case of a very short \((l \to 0)\) and lossless \((\gamma = j\beta)\) line it follows approximately:

\[ e^{-\gamma l} \approx 1 - j\varphi, \quad \varphi = \beta l. \] (27)

By inserting \(z\) into (25) we get:

\[ S \approx \left( \begin{array}{cc} \frac{1}{2z + (z - 1)^2 j\varphi} & \frac{j\varphi(z^2 - 1)}{2z} \\
\frac{2z}{j\varphi(z^2 - 1)} & \frac{2}{j\varphi} \end{array} \right). \] (28)

We now want to distinguish two cases, first if the characteristic impedance is very high in comparison to the impedance of the adjacent lines, second if the impedance is much lower:

\[ Z_0 \gg Z_1: \quad S \approx \left( \begin{array}{cc} 1 \quad 2z \\\n2z & j\varphi(z^2 - 1) \end{array} \right). \] (29)

\[ \implies \quad \text{The result is the scattering matrix of a series inductance with:} \]

\[ L = \frac{Z_0}{v_{ph}} l = L' l. \] (30)

\[ Z_0 \ll Z_1: \quad S \approx \left( \begin{array}{cc} \frac{1}{2 + j\varphi/z} & \frac{2}{j\varphi} \\
\frac{2}{j\varphi} & \frac{2}{z} \end{array} \right). \] (31)

\[ \implies \quad \text{The result is the scattering matrix of a parallel capacitance with} \]

\[ C = \frac{1}{Z_0 v_{ph}} l = C' l. \] (32)

That means if we insert into a transmission line having the impedance \(Z_1\) a short transmission line with the different impedance \(Z_0\) we get a reactive element which is formed by the inductance or capacitance per unit length.

In real transmission lines there is an additional perturbation at the interfaces between the adjoining line sections. The abrupt step in width of the outer conductor, the inner conductor, or both results in additional capacitances (cf. Fig. 7), which should be considered in the design. Additionally, the parasitic influence of the current fringing effect may be considered by another series inductance.

4.1.1 Low Pass Filter

Using such elements in a row it is possible to build \(L-C\) networks (so-called hi-lo-structures) and therefore low pass filters.
4 Filters Made Up of Short Transmission Lines

(a) Electric field pattern.

(b) Stray capacitance.

Fig. 7: Step in width of the inner conductor of a coaxial line.

Figure 8 shows the definition of the low pass prototype parameters \(g_i\). The order of the filter \(n\) and the values \(g_i\) can be depicted dependant on the necessary attenuation behaviour from a filter catalogue. The reactances are:

\[
L_i = \frac{R_0}{\omega_g} g_i, \quad C_i = \frac{1}{R_0 \omega_g} g_i, \tag{33}
\]

with the characteristic resistance \(R_0\) and the reference angular frequency \(\omega_g\). The load resistances are:

\[
R_{n+1} = g_{n+1} R_0, \quad G_{n+1} = \frac{g_{n+1}}{R_0}. \tag{34}
\]
Fig. 8: Low pass prototype parameters.
5 Questions and Problems on the Lab

Problem 1: Discuss the advantages and disadvantages of the coaxial transmission line in comparison to the most important other line types, planar transmission line and (hollow) waveguide, with respect to:

- Technical frequency range
- Losses
- Maximal power transfer
- Mechanical aspects
- Integration of lumped (concentrated) elements

Problem 2: Sketch the TEM field pattern of the fundamental mode in the coaxial transmission line in cross section and in longitudinal cut as well.

Problem 3: Which other matrix parameters than scattering parameters do you know to characterize N-ports?

Problem 4: Write down the relation between the incident and reflected amplitudes of the power waves and the voltages and currents $U$ and $I$, resp. in TEM waveguides. What is the exact definition of $a$ and $b$?

Problem 5: Which at least four methods and measuring devices do you know to get magnitude and phase of an unknown complex impedance?

Problem 6: Why is there no information about phase available using a scalar set-up?

Problem 7: Think about the calibration procedure by using the attenuator according to Fig. 2, page 3. Why the linearity doesn’t play such an important role in the calibration and measuring procedures?

Problem 8: Why should the RF power received by a detector diode not be too high?

Problem 9: How to convert $|s_{ij}|$ in decibel? What is the meaning of $-20$ dBm?

Problem 10: What is the input reflection coefficient of port 1 of a twoport in the case port 2 was matched using the characteristic impedance?

Problem 11: Write down the scattering matrix of an ideal 3 dB directional coupler (backward coupler); ports 1 and 4 as well as ports 2 and 3 should the decoupled ports.

Problem 12: Calculate the decoupling of a lossless directional coupler in dB having the coupling attenuation 10 dB and directivity 30 dB. Determine now the insertion loss in dB.

Problem 13: Determine the input impedance of a line which is connected at port 2 by $Z_2$? What is the shortest necessary line length to get an inverter behaviour?
What are the values of $K$ and $k$ in this case?

**Problem 14:** A resistance $R=12.5\,\Omega$ should be matched to $50\,\Omega$ at $5\,\text{GHz}$. Determine the necessary impedance and physical line length of the air filled transforming line ($\lambda/4$ transformer)?

**Problem 15:** Sketch the frequency response of such a $\lambda/4$-transformer designed for $5\,\text{GHz}$ and numbers from the previous problem for the frequencies $f=0$, $2.5\,\text{GHz}$; $5\,\text{GHz}$; $7.5\,\text{GHz}$ and $10\,\text{GHz}$ in a *Smith Chart* normalized to $50\,\Omega$ and concatenate the points in a reasonable way.

Between which extremal values in dB is the amplitude of the reflection in the given frequency range?

**Problem 16:** Only the parallel capacitance of a high-low low pass filter (ladder filter) in coaxial line technique, i.e. a filter made by serial high-ohmic and low-ohmic transmission lines, shall be designed. However, the transition from the transmission line acting as a capacitor to the adjacent transmission lines should be considered by additional capacitances. This transition is not part of the equivalent circuit consisting of TEM lines, only. The inner diameter of the outer conductor of the air-filled coaxial line is given by $D=7\,\text{mm}$. The characteristic impedance of a coaxial line is:

$$Z \approx \frac{60\,\Omega}{\sqrt{\varepsilon_r}} \ln \left(\frac{D}{d}\right).$$

1) Determine the characteristic impedance $Z_C$ of the short transmission line which acts as the capacitor $C=1.3\,\text{pF}$. The outer diameter of the inner conductor is given by $d_C=5.5\,\text{mm}$.

2) Determine the outer diameter of the inner conductor $d_{50}$ of the $50\,\Omega$ feeding lines.

3) Determine the necessary line length $l_C$ of the parallel capacitance $C$ in coaxial design without taking into account the adjacent transmission lines.

4) Now take into account the effect of the step in width at the transition of the $50\,\Omega$ transmission line to the capacitance section with help of Fig. 7 on page 9 and determine the necessary line length $l_C'$ of the capacitance section.

**Problem 17:** The prototype elements of a Chebyshev low pass filter of 5th order and 0.1 dB ripple are:

$$g_0 = g_6 = 1.000, \quad g_1 = g_5 = 1.147, \quad g_2 = g_4 = 1.371, \quad g_3 = 1.975$$

Determine the elements in the case $R_b=50\,\Omega$, $\omega_g = 2\pi \cdot 5 \cdot 10^9/\text{s}$ for a filter which starts with an inductance.
6 Measuring Tasks

Set-up of the RF signal source generator *Agilent N5181A*:

1) Connect the SMA plug of the data logger with the input 'TRIG IN' of the source.
2) Adjust the following parameters of the source:
   - SWEEP - Sweep - Freq On
   - SWEEP - Configure Step Sweep
     - Freq Start: 100kHz
     - Freq Stop: 6GHz
     - # Points: 1000
     - More - Step Dwell: 100us (Minimum)
   - SWEEP - More - Sweep Trigger - Ext - Trigger In Polarity → Pos
   - SWEEP - More - Point Trigger - Ext - Trigger In Polarity → Pos
   - Set-up the power:
   - AMPTD - according to the task
   - RF On
3) Hints:
   - If during the measurements the 'Current Point' is not the first point (1/1000), perform:
     - SWEEP - Single Sweep - Sweep Repeat Cont

Usage of the software used for the calibration and measurement data transfer and display

1) Log in a user 'praktikum', your tutor knows the password.
2) Change into the directory for all data. Examples:
   - cd 1_Praktikum/ehf_V04_Skalare-Spara/···
   - cd 1_Praktikum/ehf_V05_Planare_Schaltungen/···
3) Start the graphical user interface (GUI)
   - DataloggerFrontend
4) Press the following button if you want to change the virtual sheet:
   - New Sheet
5) For the storage of a new calibration curve:
   - Calibration
6) For the storage of a new measurement curve (differs from the calibration curve only in color and line type):
   - Measurement
7) Display of all curves and printing using 'gv':
8) Please label the curves immediately after printing.

Hints:

- All files (data files, postscript files and so on) are stored in the same directory where the program was invoked from. These files have unique numbers referring to the virtual sheets, and it is always possible to rename or to delete some of them.
- If the GUI was terminated and started again the file number of the following measured data are increased automatically.

Task 1: Take the calibration curves (transmission calibration), but you have to distinguish which RF generator you are using:

Agilent N5181A (up to 6 GHz):
from 5 dBm ≈ 0 dB in 2 dB steps until −37 dBm ≈ −42 dB.

Agilent N5183A (up to 32 GHz):
from 20 dBm ≈ 0 dB in 2 dB steps until −22 dBm ≈ −42 dB;
make use of a 20 dB attenuator at the output port of the generator to avoid the destruction of the detector diode.

Measure $|s_{21}|$, $|s_{31}|$ and $|s_{41}|=|s_{32}|$ of the directional coupler.

Calculate and determine the coupling attenuation and the directivity using the measured results.

Task 2: Perform a reflection calibration using the same values for the calibration curves.

Measure $|s_{11}|$

- of a load resistance (professional manufactured) and
- a detector (without the digital wiring at the output).

Tell the difference between the results.

Explain the influence of a poorly matched detector on the measuring result.

Measure $|s_{11}|$ of the “lumped” elements using the same calibration sheet:

- Metal film resistance (50 Ohm) in a coaxial shielding,
- SMD resistance (51 Ohm) (SMD=Surface Mounted Device),
- SMD capacitance (10 pF),

and discuss the results.

Task 3: Perform a new reflection calibration.

Measure $|s_{11}|$ of the $\frac{1}{4}$-transformer including the already inserted load.

Measure $|s_{11}|$ and $|s_{21}|$ of the coaxial low pass filter using the same calibration sheet.

Make use of the reflection calibration for both measurements.
Smith Chart

Reflexionsfaktor / reflection coefficient

Aufgabe / problem:

$Z_0 =$  $\Omega$
6 Measuring Tasks