Lab course **RF Engineering**
Lab 7: Scalar Scattering Parameter Meas. (waveguide)

Name: ........................ Date: ........................

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Please note the Important Hints on a separate sheet.

1 Introduction

This lab deals with scattering parameter measurements of X-band (8.2 GHz–12.4 GHz) rectangular waveguides. The basic knowledge of scattering parameters was treated in lab Scalar scattering parameter measurements (coaxial).

Read the appropriate sections of the mentioned script (once more) as preparation for the oncoming text. The content is presumed as well-known.

In contrast to the coaxial line, the rectangular waveguide consists of a single metal structure which needs a special field theoretical treatment used for these kinds of transmission lines. In the following sections the waveguide properties and its connection to the scattering parameters will be described in details.

2 Transmission Lines

In a waveguide in which the cross section as well as the material parameters don’t change in the direction of propagation and if there are more than one metallisation or the frequency is high enough, waves are able to travel along this waveguide. The field pattern in the cross section depends on the geometry of the waveguide, only. In a first attempt assume an arbitrary waveguide.

All waves have to fulfill Maxwell’s equations:

\[
curl \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t},
\]

(1)

\[
curl \vec{H} = \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}.
\]

(2)

Without restrictions we assume a harmonic time dependence of the waves. A wave travelling in \( z \)-direction may be written as

\[
\vec{E} = \vec{E}(x, y) e^{j\omega t - jk_z z},
\]

(3)

\[
\vec{H} = \vec{H}(x, y) e^{j\omega t - jk_z z},
\]

(4)

where \( k_z \) is the wave number in the propagation direction.

Applying \( \frac{\partial}{\partial z} = -jk_z \) and \( \frac{\partial}{\partial t} = j\omega \) we get

\[
-\frac{\partial E_z}{\partial y} - jk_z E_y = j\omega \mu H_x,
\]

(5)

\[
\frac{\partial E_z}{\partial x} + jk_z E_x = j\omega \mu H_y,
\]

(6)
\[-\frac{\partial E_y}{\partial x} + \frac{\partial E_x}{\partial y} = j\omega \mu H_z, \quad (7)\]
\[\frac{\partial H_z}{\partial y} + jk_z H_y = j\omega \epsilon E_x, \quad (8)\]
\[-\frac{\partial H_z}{\partial x} - jk_z H_x = j\omega \epsilon E_y, \quad (9)\]
\[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega \epsilon E_z. \quad (10)\]

The field components in the waveguide cross section \((E_x, E_y, H_x, H_y)\) may be expressed after rewriting (5) to (10) as functions of \(H_z\) and \(E_z\), resp.:
\[E_x = -\frac{j}{k_p^2} \left( k_z \frac{\partial E_z}{\partial x} + \omega \mu \frac{\partial H_z}{\partial y} \right), \quad (11)\]
\[E_y = \frac{j}{k_p^2} \left( k_z \frac{\partial E_z}{\partial y} - \omega \mu \frac{\partial H_z}{\partial x} \right), \quad (12)\]
\[H_x = -\frac{j}{k_p^2} \left( k_z \frac{\partial H_z}{\partial x} - \omega \epsilon \frac{\partial E_z}{\partial y} \right), \quad (13)\]
\[H_y = -\frac{j}{k_p^2} \left( k_z \frac{\partial H_z}{\partial y} + \omega \epsilon \frac{\partial E_z}{\partial x} \right), \quad (14)\]

and the field components in the propagation direction have to fulfill the reduced wave equations (Helmholtz equations to be more precise since it is written in the frequency domain):
\[\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + k_p^2 E_z = 0, \quad (15)\]
\[\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + k_p^2 H_z = 0, \quad (16)\]

with the transversal eigenvalue \(k_p^2 = \omega^2 \mu \epsilon - k_z^2\).

Considering the resulting wave in the waveguide as a sum of partial waves, which are reflected at the waveguide boundaries, \(k_p\) may be understood as the propagation constant in the lateral direction. Thus, the wave travels in a zigzag course from one sidewall to the other sidewall.

To determine the field distribution in the cross section the solution of the two-dimensional Helmholz equation of \(H_z\) and \(E_z\), (15), (16), resp., is needed.

### 2.1 Classification of Modes

In a first step the electromagnetic waves should be classified in three groups:

1) **TEM waves**

There are no electromagnetic components in the longitudinal direction, i.e. \(E_z = 0\) and \(H_z = 0\) and the wave number in lateral direction \(k_p\) is zero.

2) **TE waves**

TE waves have a longitudinal magnetic field component and a pure Transversal Electric field pattern.
3) **TM waves**

TM waves have a longitudinal electrical field component and a pure transversal magnetic field pattern.

Thus, the TEM wave is a special case of the TE and TM waves.

In waveguides in which the cross section is a single connected area, TEM modes are not able to propagate. This will be seen in the following section:

![Transmission lines with two and one conductor](image)

Fig. 1: Transmission lines with two and one conductor.

The second Maxwell equation in integral form is:

\[
\oint_{(A)} \vec{H} \cdot d\vec{s} = \frac{\partial}{\partial t} \int_{A} \vec{D} \cdot d\vec{A} + \int_{A} \vec{J} \cdot d\vec{A}.
\]  

(17)

(A) is the edge of the area A. Since the magnetic field is purely transversal, \( \text{div}\vec{B}=0 \) in the cross section is valid. Because of this condition we are able to find a closed flux line in the plane where the integral \( \oint_{(A)} \vec{H} \cdot d\vec{s} \) does not vanish. On the right hand side of (17) at least one of the two integrals must not be zero. But since there is no inner conductor, the integral on the current has to be zero. The same is true for the integral of the displacement current in direction of propagation, since the electric field is according to the precondition purely transversal. This leads to a contradiction, i.e. there are no TEM waves propagating in the waveguide.

### 2.2 TE Modes in a Rectangular Waveguide

The solution of the system of differential equations leads in the case of a rectangular waveguide according to figure 2 to a closed expression.

In this section the solution of waves having a transversal field pattern should be considered, i.e. we need the solution of the Helmholtz equation of the longitudinal magnetic field component. The Separation set-up

\[
H_z = X(x)Y(y)e^{j\omega t-jk_x z}
\]  

(18)

leads by considering the boundary condition \( E_{\text{tan}}=0 \) on the waveguide border to the following equations, which are fixed beside the coefficient \( C_{\text{TE}} \). Without the term \( e^{j\omega t-jk_x z} \), we have

\[
E_z = 0,
\]

(19)
\[ H_z = C_{\text{TE}} \cos(k_x x) \cos(k_y y), \] (20)
\[ E_x = \frac{j \omega \mu}{k_x^2 + k_y^2} C_{\text{TE}} \cos(k_x x) \sin(k_y y), \] (21)
\[ E_y = \frac{j \omega \mu}{k_x^2 + k_y^2} C_{\text{TE}} \sin(k_x x) \cos(k_y y), \] (22)
\[ H_x = -\frac{j k_x k_z}{k_x^2 + k_y^2} C_{\text{TE}} \sin(k_x x) \cos(k_y y), \] (23)
\[ H_y = -\frac{j k_y k_z}{k_x^2 + k_y^2} C_{\text{TE}} \cos(k_x x) \sin(k_y y), \] (24)

with
\[ k_x = \frac{m \pi}{a} \quad m \in \{0, 1, 2, 3, \ldots\}, \] (25)
\[ k_y = \frac{n \pi}{b} \quad n \in \{0, 1, 2, \ldots\}, \] (26)

without \( m = n = 0, \) (27)

and the dispersion relation (separation equation)
\[ k^2 = \left( \frac{\omega}{c} \right)^2 = k_x^2 + k_y^2 + k_z^2 = \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 + k_z^2. \] (28)

The wave number in propagation direction \( k_z \) is
\[ k_z = \sqrt{\left( \frac{\omega}{c} \right)^2 - \left( \frac{m \pi}{a} \right)^2 - \left( \frac{n \pi}{b} \right)^2} \] (29)

with
\[ jk_z = \gamma = \alpha + j \beta \quad c = \frac{c_0}{\sqrt{\varepsilon_r \mu_r}}. \] (30)

Therefore the solution consists of different partial solutions which differ in \( m \) and \( n \) and are called modes.
According to (29) \( k_z \) is purely negative imaginary below the special frequency, the \textbf{cut-off frequency}, and this frequency grows with \( m \) and \( n \). At frequency above these cut-off frequencies the appropriate modes are propagating, at frequencies below the mode is damped aperiodically and the whole power is reflected. The mode having the smallest cut-off frequency (for \( a>b \) which is true for all common rectangular waveguide types) is the \( \text{TE}_{10} \) mode (\( m=1 \) und \( n=0 \)), called \textbf{fundamental mode}. Please note, there is no \( \text{TE}_{00} \) mode available.

### 2.3 TM Modes in Rectangular Waveguide

By solving the Helmholtz equation for the \( E_z \) component to get the solutions for the TE modes, the dispersion relation is the same, but there are only modes having the indices \( m>0 \) and \( n>0 \).

### 2.4 TE\(_{10}\) Mode in Rectangular Waveguide

The mode having the lowest cut-off frequency (\( \text{TE}_{10} \) mode) is of special technical interest. Using \( e^{-jk_z z} \) as \( z \)-dependence of the incident wave and \( e^{+jk_z z} \) as \( z \)-dependence of the reflected wave, renaming the coefficient from \( C_{\text{TE}} \) to \( a_{10} \) for the incident and to \( b_{10} \) for the reflected wave, we get the following field components of the \( \text{TE}_{10} \) mode (again without the time dependence \( e^{j\omega t} \))

\[
E_z = 0, \\
H_z = -j \frac{\pi}{\sqrt{\mathcal{Z}_H \beta}} \cos \left( \frac{\pi x}{a} \right) \left( a_{10} e^{-j\beta z} + b_{10} e^{j\beta z} \right), \\
E_x = 0, \\
E_y = \sqrt{\mathcal{Z}_H} \sqrt{\frac{2}{ab}} \sin \left( \frac{\pi x}{a} \right) \left( a_{10} e^{-j\beta z} + b_{10} e^{j\beta z} \right), \\
H_x = -\frac{1}{\sqrt{\mathcal{Z}_H}} \sqrt{\frac{2}{ab}} \sin \left( \frac{\pi x}{a} \right) \left( a_{10} e^{-j\beta z} - b_{10} e^{j\beta z} \right), \\
H_y = 0,
\]

with the phase constant \( \beta = k_z \) and the \textbf{wave impedance} or \textbf{field characteristic impedance} \( \mathcal{Z}_H = \frac{\omega \mu}{\beta} \).

In Fig. 3 the field pattern in the cross section and on the horizontal waveguide boundary is shown.

1) \textbf{Frequency range}

The X-band waveguide width is \( a=22.86 \text{ mm} \) and height \( b=10.16 \text{ mm} \). This leads for the \( \text{TE}_{10} \) mode with

\[
\beta = \sqrt{\left( \frac{\omega_c}{c} \right)^2 - \left( \frac{\pi}{a} \right)^2} = 0 \tag{31}
\]
to the cut-off frequency
\[ \frac{\omega_c}{2\pi} = f_c = \frac{c}{2a} = 6.57 \text{ GHz}. \] (32)

The first higher order mode TE\(_{20}\) has the cut-off frequency
\[ f_c = \frac{c}{a} = 13.14 \text{ GHz}. \] (33)

In the frequency range 6.57 GHz–13.14 GHz there is only the TE\(_{10}\) mode propagating and ensures a unique (single mode) power transfer. As technically reasonable frequency range of the X-band waveguide, the range
\[ 1.25f_c < f < 1.9f_c \] (34)
is chosen. First, avoiding the very dispersive frequency range just above of its cut-off frequency, second, to ensure a frequency distance to the cut-off frequency of the first higher mode. In the case this higher mode is exciting still below its cut-off frequency, the fields are damped exponentially, but the decay is slower and therefore the influence on nearby parts of the waveguide circuit (if there are any) is more important with increasing frequency.

2) **Dispersion**

The dispersion relation provides the function of the phase constant \(\beta\) on frequency; this relation is linear in case of TEM waves. The dispersion of the TE\(_{10}\) wave is in the considered frequency range quite large. The **phase velocity** is
\[ v_{ph} = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2}}, \] (35)

and the **group velocity** is
\[ v_{gr} = \frac{\partial \omega}{\partial \beta} = \frac{c^2}{v_{ph}} = \frac{c^2}{\omega} \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2}. \] (36)
The group velocity is responsible for power transfer. At frequencies decreasing from higher frequencies to the cut-off frequency, the group velocity decreases and vanishes in the end, and therefore no power transport is possible.

3) **Power of the TE\textsubscript{10} mode**

The scattering parameters introduced in the fundamental lectures were normalized to the characteristic impedance or power traveling along the transmission line. The power in the waveguide was summed up of the incident and reflected waves

\[
P_{ij} = \frac{1}{2} \left( |a_{ij}|^2 - |b_{ij}|^2 \right). \quad (37)
\]

\(a_{ij}\) and \(b_{ij}\) are called wave amplitudes. In the following the relation of the scattering parameters and the field amplitudes of the TE\textsubscript{10} wave is shown. The power transported by a wave is determined by the **Poynting vector**

\[
\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* . \quad (38)
\]

The power through the waveguide cross section is therefore

\[
P_z = \frac{1}{2} \int_0^a \int_0^b \left( \vec{E} \times \vec{H}^* \right)_z \, dy \, dx . \quad (39)
\]

In case of the TE\textsubscript{10} wave the integral is reduced to

\[
P_z = \frac{1}{2} \int_0^a \int_0^b -E_y H_x^* \, dy \, dx ,
\]

\[
= \int_0^a \int_0^b \frac{1}{ab} \sin^2 \left( \frac{\pi}{a} x \right) \left( a_{10} e^{-j \beta z} + b_{10} e^{j \beta z} \right) \left( a_{10} e^{-j \beta z} - b_{10} e^{j \beta z} \right)^* \, dy \, dx ,
\]

\[
= \frac{1}{2} \left( |a_{10}|^2 - |b_{10}|^2 \right) . \quad (40)
\]

The fields were renormalized in section 2.4 on page 5 in such a way, that the coefficients \(a_{10}\) and \(b_{10}\) were equivalent to the wave parameters. This normalization is analogue to the normalization by the characteristic impedance which was done in the case of TEM waves calculations.

4) **Attenuation in the vicinity of the cut-off frequency**

In the case the operation frequency is below cut-off, the wave is damped aperiodically and the propagation constant \(\gamma\) is purely real. We get for the TE\textsubscript{10} mode

\[
\beta = j \sqrt{ \left( \frac{\pi}{a} \right)^2 - \left( \frac{\omega}{c} \right)^2 } = j \alpha ,
\]

\[
\Rightarrow E_x(z) = E_x(z=0) e^{-\alpha z}.
\]
3 Waveguide Discontinuities

The attenuation $a$ is

$$P \propto |E|^2, |H|^2,$$

$$|s_{21}|^2 = \left| \frac{E_x(z)}{E_x(z=0)} \right|^2 = e^{-2\alpha z} \quad \text{(e.g.)},$$

$$a = \frac{1}{|s_{21}|^2} = 10 \log_{10} e^{2\alpha z} \text{dB} = 20\alpha z \log(e) \text{dB},$$

$$z=1 \text{m} : \quad a = 20\alpha \log(e) \frac{\text{dB}}{\text{m}}.$$

3 Waveguide Discontinuities

A discontinuity in a waveguide may be characterized by two-port scattering parameters calculated using numerical methods. In perfectly bounded waveguides having rectangular shapes the mode matching technique (MMT) (orthogonal expansion) is very well suited. The discontinuity structure is divided in homogenous waveguide sections, and in each section a complete set of modes has to be set-up. Fulfilling the continuity conditions of all tangential electromagnetic components at the section interfaces leads to the scattering parameter representation of the two-port. Equivalent circuits as a description of discontinuities may be used to get approximate scattering parameters as well. In this case, the waveguide is replaced by a transmission line having a characteristic impedance. In the strict sense there are no characteristic impedances in non-TEM transmission lines and a possible definition is not unique. But it may be defined by the ratio

- of one voltage $U = -\int \vec{E} \, ds$ and a current flowing through a suited cross section,
- of transported power by square of the current,
- of square of one voltage by transported power.

The integration line has to be chosen carefully; in the rectangular waveguide the definition by voltage and power is applied. In this lab only waveguide sections of the same cross section are used; therefore only the impedance ratios of the different sections are of interest.

3.1 Equivalent Circuits

Thin metallic irises in the waveguide cut in the homogenous cross section and generate reflections. For this kind of discontinuities there are equivalent circuits based on the transmission line theory and which consist of a few lumped elements, only. With the aid of these equivalent circuits it is much easier to design waveguide filters etc. as by using the “exact” method described above or other numerical methods.

In figure 4 three equivalent circuits are shown as an example: the abrupt reduction of the waveguide height $b$, the abrupt reduction of the waveguide width $a$, and a small hole in a otherwise perfect short circuit.

Appropriate approximation formulas—i.e. reactance $X$ and susceptance $B$ as function of geometry parameters and frequency—may be found in, e.g.
As an example, the scattering parameters of a shunt susceptance are

\[ s_{11} = s_{22} = \frac{-jB/Y_0}{2 + jB/Y_0}, \] \hspace{1cm} (45)

\[ s_{12} = s_{21} = \frac{2}{2 + jB/Y_0}. \] \hspace{1cm} (46)

### 3.2 Resonator with Lumped Elements

By using an LC resonance circuit and two resistances a wired resonator according to Fig. 5 may be build up, characterized by the resonance frequency and the 3dB bandwidth. The extracted results in this section will then be applied to the waveguide resonator.

The input impedance of the circuit is

\[ \frac{U}{I} = Z = 2R + j\left(\omega L - \frac{1}{\omega C}\right) = 2R + jX_R \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \] \hspace{1cm} (47)

with the image impedance \( X_R = \sqrt{\frac{L}{C}} \) \hspace{1cm} (48)

and the resonance angular frequency \( \omega_0 = \frac{1}{\sqrt{LC}} \). \hspace{1cm} (49)

The Taylor series approximation up to the linear term of the input impedance \( Z \) at \( \omega \approx \omega_0 \) leads to

\[ Z \approx 2R + jX_R \frac{2(\omega - \omega_0)}{\omega_0}. \] \hspace{1cm} (50)
The 3 dB edge frequencies may be calculated from this equation

\[
\frac{R\omega_0}{X_R} = |\omega_{3\text{dB}} - \omega_0|
\]  

(51)

and the bandwidth

\[
\Delta\omega = 2\frac{R\omega_0}{X_R} .
\]  

(52)

The quality factor of the resonance circuit is defined by

\[
Q := \frac{\omega_0}{\Delta\omega} = \frac{X_R}{2R} .
\]  

(53)

In waveguide technique single-circuit resonators may be synthesized by using \( \frac{\lambda}{2} \) resonators with irises as impedance or admittance inverters as coupling elements between the feeding transmission lines and the resonator.

### 3.3 Irises as Inverters

The input impedance of an inverter loaded by \( Z_a \) at the second port is

\[
Z_e = k^2 \frac{Z_L^2}{Z_a} .
\]  

(54)

\[\begin{array}{c}
\includegraphics[width=0.5\textwidth]{transmission_line_with_shunt_susceptance.png}
\end{array}\]

Fig. 6: Transmission line with shunt susceptance.

Figure 6 shows a transmission line and a shunt susceptance in the middle of the line. This structure behaves like an inverter, if the conditions

\[
b = B Z_L = \pm \frac{1 - k^2}{k} ,
\]  

\[
\tan \varphi = \frac{2}{b} , \quad \varphi = \beta l_i
\]  

(55)

(56)

are fulfilled with the normalized inverter constant \( k \).

In the case of an inductive reactance in the waveguide, the transmission line length \( l_i \) becomes negative; using this type of realization the periodicity of the electrical behaviour of transmission lines has to be exploited to get reasonable line lengths.
3.4 Waveguides with Irises

A waveguide resonator may be synthesized by using two irises, where the coupling between the feeding lines and the resonator is done by inverters. In figure 7 the equivalent circuit of such a structure is shown.

![Equivalent circuit of the waveguide resonator with irises.](image)

The irises act as inverters and transform the load impedance $Z_L$ to the normalized load (equation (54) and $Z_e=R_e$, $Z_a=Z_L$):

$$r = \frac{R}{Z_L} = k^2 \ll 1. \quad (57)$$

The transmission line with length

$$l_r = p \frac{\lambda}{2}, \quad p \in \mathbb{Z}$$

is therefore—appropriate choice of $k^2$ assumed—almost short circuited at both ends and acts as a resonator. We get the resonance characteristic from the input impedance of a short circuited line

$$z_K = \frac{Z_K}{Z_L} = j \tan \left( \frac{2\pi l}{\lambda_g} \right). \quad (58)$$

The wavelength $\lambda_g$ in the waveguide is

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left( \frac{\lambda}{\lambda_c} \right)^2}}, \quad (59)$$

with the cut-off wavelength in free space $\lambda_c$ and the wavelength in free space $\lambda$.

With $c=f\lambda=f_c\lambda_c$ (58) becomes

$$z_k = j \tan \left( \frac{2\pi}{\lambda} \sqrt{1 - \left( \frac{\lambda}{\lambda_c} \right)^2} \right) = j \tan \left( \frac{2\pi f}{c} \sqrt{1 - \left( \frac{f_c}{f} \right)^2} \right), \quad (60)$$

$$z_k = j \tan \left( \frac{2\pi}{c} \sqrt{f^2 - f_c^2} \right). \quad (61)$$

In the resonance case (index res), i.e. $f=f_{\text{res}}$, $l_r$ is roughly a multiple of the half wavelength, i.e. $l_r=p\frac{\lambda_{g,\text{res}}}{2}$, (61) may then be approximated by a Taylor series expansion at frequency $f \approx f_{\text{res}}$:

$$z_K \approx z_k|_{f=f_{\text{res}}} + \frac{\partial z_K}{\partial f}|_{f=f_{\text{res}}} (f - f_{\text{res}}), \quad (62)$$
\[ z_k \approx 0 + j \frac{1}{\cos^2 \left( \frac{2\pi}{c} \sqrt{f^2 - f_0^2} \right)} \left| \frac{2\pi}{c} \frac{1}{2} \frac{2f}{\sqrt{f^2 - f_0^2}} \right| \ (f - f_{\text{res}}), \quad (63) \]

\[ z_k \approx j2\pi \frac{1}{c} \left| \frac{1}{\sqrt{1 - \left( \frac{f}{f_{\text{res}}} \right)^2}} \right| \ (f = f_{\text{res}}), \quad (64) \]

\[ z_k \approx j2\pi \frac{\lambda_{g,\text{res}}}{2} \frac{1}{\lambda_{\text{res},f_{\text{res}}}} \frac{\lambda_{g,\text{res}}}{\lambda_{\text{res}}} (f - f_{\text{res}}), \quad (65) \]

\[ z_k \approx j \frac{\pi}{2} \left( \frac{\lambda_{g,\text{res}}}{\lambda_{\text{res}}} \right)^2 \frac{2(f - f_{\text{res}})}{f_{\text{res}}}, \quad (66) \]

A comparison of (66) with (50) shows the equivalence of a discretely built resonance circuit and a transmission line resonator in the vicinity of the resonance frequency \( f_{\text{res}} \) by the image impedance

\[ x_{\text{res}} = \frac{X_{\text{res}}}{Z_L} = \frac{1}{Z_L} \sqrt{\frac{L}{C}}. \quad (67) \]

The transmission line acts as a series connection \( x_{\text{res}} = \frac{\omega_0 L}{Z_L} \) with the additional source and load resistances \( R \) (figure 8).

\[ U \quad L \quad C \quad Z_L \]

\[ R \ll Z_L \]

Fig. 8: \( LC \) series resonance circuit and transmission line equivalent circuit.

With this the quality factor \( Q \) and the bandwidth \( \Delta \omega \) are

\[ Q = \frac{x_{\text{res}}}{2r} = \frac{p \pi}{2} \left( \frac{\lambda_{g,\text{res}}}{\lambda_{\text{res}}} \right)^2 = \frac{\omega_{\text{res}}}{\Delta \omega}. \quad (68) \]
4 Waveguide Circulator

The 3-port waveguide circulator usually consists of a 120° H-plane junction having a ferrite post at the center. The ferrite is an anisotropic material and thus a circulator is a non-reciprocal component.

A biasing magnetic field (DC-magnetization) is applied in the direction of the cylindrical axis of the ferrite post.

To easily understand the principle of operation of the circulator, it is convenient to assume that the rectangular waveguides operate in the dominant TE$_{10}$ mode and that the diameter of the ferrite post is much smaller than the waveguide width.

The circulator can be described as a cylindrical cavity which is coupled to branching waveguides, for example by means of small irises. At the operating frequency, the TM$_{110}$ exists in the cavity. The TM$_{110}$ can be also described as a dipole mode. The electric field increases approximately linearly with the radius and the field is zero longitudinal on the axis. Such a standing wave pattern can be generated by the superposition of a left-handed and a right-handed pattern. When the DC-magnetization is applied, the two patterns do not resonate anymore at the same frequency. In particular, the two patterns have different propagation constant and phase velocities.

If the operating frequency and the bias of the circulator are set properly, so that the phase angles of the impedances of the two modes are each 30°, than the standing wave pattern can be rotated by 30° from the original configuration.

Figure 9 shows the top view of a waveguide circulator, where in the center point of the symmetric 3-port branching a ferrite post was inserted.

![Fig. 9: E- and H-field patterns in a waveguide circulator.](image)

In the left drawing there is no DC-magnetization or a DC-magnetization which doesn't fit to the structure/frequency, the field pattern is unchanged in comparison to the branching without the ferrite post. In the right drawing a ferrite was pre-magnetized in z-direction—perpendicular to the H-plane of the waveguide—which results in a turning of the field as described just before. In the presented example port 2 is decoupled; on the other hand, in
the ideal case without DC-biasing, the whole power would travel from port 1 to port 3. In a 3-port waveguide circulator two ports can be coupled to the standing-wave pattern and the third port lies at the null of the field. The transmission is given by the port order (in this case 1–3–2–1).

Circulators are used in transmission/receiving systems (TR systems) to decouple the transmission from the receiving path.

5 Measurement Set-up

Figure 10 shows the measurement set-up.

![Figure 10: Measurement set-up.](image)

6 Transition Waveguide–Coaxial Line

For the purpose of coupling out of power there is a need of transitions from waveguide to coaxial line.

In Fig. 11 this kind of transition is shown, where the coaxial inner conductor immerses into the waveguide. The coupling is done by the capacitive stick and is called capacitive coupling.

In a distance of \(\frac{\lambda}{4}\) behind this probe head there is a short circuit (back short), which leads to the maximum of the voltage (or to be more precise: electric field) at the position of the probe head.
Fig. 11: Transition from the rectangular waveguide (left figure on the left side) to the coaxial cable (connector type N, top).
7 Questions and Problems on the Lab

The questions are—if nothing else is indicated—related to the TE_{10} mode.

**Problem 1:** Which quantity cannot be measured using a scalar measuring set-up?

**Problem 2:** What is the meaning of matching?

**Problem 3:** Write down the general scattering matrix of a threeport.
Which physical meaning has \( s_{32} \)?

**Problem 4:** Which relation do you know about the amplitudes and phases of the scattering parameters in a lossless and reciprocal twoport?
Write down the unitary condition and assume \( s_{ik} = |s_{ik}|e^{j\phi_{ik}} \).

**Problem 5:** What are the cut-off frequencies of TEM modes?

**Problem 6:** Write down the equation for the wavelength of a TE_{mn} or TM_{mn} wave in an ideal rectangular waveguide as function of the frequency, the material parameters and the dimensions of the waveguide.

**Problem 7:** Calculate the wavelength in the waveguide at cut-off frequency.

**Problem 8:** Determine the cut-off wavelength of the TE_{10} mode, i.e. the free-space wavelength at the cut-off frequency as function of the waveguide's dimensions.

**Problem 9:** Determine the cut-off frequency of the TE_{10} mode as function of waveguide width \( a \).

**Problem 10:** Calculate the theoretical values of the attenuation constant \( \alpha \) of the TE_{10} wave at
\( f/\text{GHz}=(8.2, 9.0, 9.8, 10.6, 11.4, 12.2) \), in the case the waveguide width is \( a=14\text{mm} \).
Calculate from this the attenuation of the wave on a waveguide with length \( l=15\text{mm} \) in dB. Calculate the cut-off frequency \( f_c \) of the fundamental mode.

**Problem 11:** Write down the scattering matrix of a shunt susceptance.

**Problem 12:** Calculate the necessary normalized shunt susceptances \( b \) and inverter lengths \( l \) to get the normalized inverter constants \( k=1 \) and \( k=0.1 \) (both cases: TE_{10} mode, \( a=22.86\text{mm}, f=10\text{GHz} \)).

**Problem 13:** Calculate the normalized inverter constant \( k \) if the quality factor is given.

**Problem 14:** Write down the circuit symbol and the scattering matrix of an ideal circulator.

**Problem 15:** Which field components are coupled out by the probe head at the coaxial-waveguide transition?

**Problem 16:** Why the distance between the probe head and the short circuit is \( \frac{\lambda}{4} \) in the coaxial-waveguide transition?
Please bring along a pair of compasses.
8 Measuring Tasks

Task 1: In section 7 the attenuation constant and the cut-off frequency of a waveguide with \( a=14 \text{ mm} \) was calculated at different frequency points. The attenuation constant should now be determined by measurements using a waveguide section of 15 mm length.

1) Perform a transmission calibration.
   \((0.2 \frac{V}{cm}, \text{not unlevled } P, \{0, 3, 6, 9, 12, 15, 20, 25, 30\} \text{ dB})\)
2) Measure the attenuation versus frequency and mark the frequency points \( f/\text{GHz}=\{8.2, 9.0, 9.8, 10.6, 11.4, 12.2\} \).
3) Calculate the attenuation constant and compare these measured values with the theoretical ones and write down the values.

Task 2: Determine the shunt admittance, or shunt susceptance to be more precise, of a round iris.

1) Perform a reflection calibration up to \(-3 \text{ dB}\).
   \((0.1 \frac{V}{cm}, \{0, 0.5, 1, 1.5, 2, 2.5, 3\} \text{ dB})\)
2) Measure \(|s_{11}|\) of the round waveguide iris and determine the normalized iris susceptance \(BZ_L\) at 10.2 GHz using a Smith-Chart.
3) When is this method very inaccurate?

Task 3: Measure the waveguide filter consisting out of two round irises as inverters and a section of a X-band waveguide (resonator, \( a=22.86 \text{ mm} \)).

1) Perform a transmission calibration. Set-up:
   \((0.2 \frac{V}{cm}, \{0, 1, 2, 3, 6, 10, 15, 20, 25, 30\} \text{ dB})\)
2) Chain two irises and one waveguide section to build a single-circuit filter.
3) Measure the transmission curve.
4) Perform a reflection calibration.
   \((0.2 \frac{V}{cm}, \{0, 3, 6, 10, 15, 20\} \text{ dB})\)
5) Measure the reflection factor.
6) Determine from the 3 dB points the normalized iris susceptance \((p=1)\).

Task 4: Measurement of a circulator. Reuse the calibration of the previous task.

1) Measure \(|s_{11}|\) of the circulator.
2) Measure \(|s_{21}|\) of the circulator. Give the necessary wiring for port 3.
3) Measure \(|s_{31}|\) of the circulator. Give the necessary wiring for port 2.
4) Connect port 2 to a short circuit and measure the power at port 3.
5) Determine the amplitudes of the circulator's scattering parameters. Is the circuit lossless? Give reason for your answer.
6) What is the order of the circulator ports?
The following tasks should be performed virtually only. That means please write down the results without real measurements.

7) Match port 2 using a perfect absorber.
   What is the name of the resulting component? Is it lossless? How big is the influence of the mismatching of the load?

8) Connect the bandpass at port 2. The bandpass is ideally matched at the other port. Which transfer behaviour of this circuit do you expect?

9 Measuring Set-Up

The measuring set-up consists out of:

- 1 Wobbelgenerator / Sweep-Oszillator
- 1 Koaxialleitung (N-Verbinder) / coaxial line with N type connectors
- 1 Übergang Koaxial-(N-Verbinder) auf Hohlleiter / Transition coaxial line (N type) to waveguide
- 1 variables Dämpfungsglied / adjustable attenuator
- 1 Richtkoppler mit 10 dB Koppeldämpfung und 30 dB Richtdämpfung / directional coupler with 10 dB coupling and 30 dB directivity
- 2 Übergänge Koaxial-(SMA-Verbinder) auf Hohlleiter / Transitions coaxial (SMA type) to waveguide
- 2 Detektoren in koaxialer Bauform / detectors in coaxial design
- 2 Logarithmische Verstärker / logarithmic amplifiers
- 1 Zweikanal-Oszilloskop / 2-channel oscilloscope
- 1 Kurveneschreiber (Plotter) / plotter

Before each measurement the calibration curves have to be stored by using a plotter.