A New Interpretation of the Spectral Domain Immittance Matrix Approach

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Abstract— **A modified interpretation of the spectral domain immittance matrix approach based on the transformation of the principal axes for the spectral domain TE-and TM-field components is presented.**

I. Introduction

THE SPECTRAL DOMAIN METHOD [1], [2] has proven to be a valuable tool calculating planar transmission line properties as well as discontinuities. Vector potentials $\Phi\vec{e}_y$ and $\Psi\vec{e}_y$ for TM- and TE-field types with respect to the y-axis are set up in the different layers of the structure of interest (Fig. 1).

The potentials must satisfy the wave equation

$$\Delta\begin{Bmatrix}\Phi\\\Psi\end{Bmatrix}+k^2\begin{Bmatrix}\Phi\\\Psi\end{Bmatrix}=0, \tag{1}$$

and the field components can be calculated as follows:

$$\vec{E}=\nabla\times(\Psi\vec{e}_y)-\frac{1}{j\omega\mu}\nabla\times\nabla\times(\Phi\vec{e}_y) \tag{2a}$$

$$\vec{H}=\nabla\times(\Phi\vec{e}_y)+\frac{1}{j\omega\varepsilon}\nabla\times\nabla\times(\Psi\vec{e}_y). \tag{2b}$$

Applying a Fourier transformation, the wave equation is transformed into a simple differential equation

$$\frac{\partial^2}{\partial y^2}\begin{Bmatrix}\tilde{\Phi}\\\tilde{\Psi}\end{Bmatrix}+(k^2-\alpha^2-\beta^2)\begin{Bmatrix}\tilde{\Phi}\\\tilde{\Psi}\end{Bmatrix}=0, \tag{3}$$

where α is the Fourier variable with respect to the x-coordinate, and β may be either the propagation constant of a transmission line or the Fourier variable with respect to the z-axis. Solutions based on exponential or hyperbolic functions of this equation can be found analytically. Equally, a Fourier transform is applied to (2) to calculate the field components resulting in the following:

$$\tilde{E}_x=-\frac{\alpha}{\omega\varepsilon}\frac{\partial\tilde{\Phi}}{\partial y}-j\beta\tilde{\Psi} \tag{4a}$$

$$\tilde{E}_y=\frac{1}{j\omega\varepsilon}(\alpha^2+\beta^2)\tilde{\Phi} \tag{4b}$$

$$\tilde{E}_z=-\frac{\beta}{\omega\varepsilon}\frac{\partial\tilde{\Phi}}{\partial y}+j\alpha\tilde{\Psi} \tag{4c}$$

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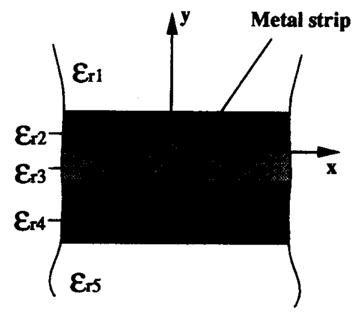

Fig. 1. Typical structure of a planar circuit to be solved with the spectral domain method.

$$\tilde{H}_x=j\beta\tilde{\Phi}-\frac{\alpha}{\omega\mu}\frac{\partial\tilde{\Psi}}{\partial y} \tag{4d}$$

$$\tilde{H}_y=\frac{1}{j\omega\mu}(\alpha^2+\beta^2)\tilde{\Psi} \tag{4e}$$

$$\tilde{H}_z=-j\alpha\tilde{\Phi}-j\frac{\beta}{\omega\mu}\frac{\partial\tilde{\Psi}}{\partial y}. \tag{4f}$$

Applying the boundary conditions for the field components at the interfaces between the different layers then leads to the following equations ($y=0$, Fig. 1) where $\tilde{J}_x, \tilde{J}_z$ are the surface current densities in the metallization layer:

$$\begin{aligned}\tilde{E}_x&=Z_{xx}\tilde{J}_x+Z_{xz}\tilde{J}_z\\\tilde{E}_z&=Z_{xz}\tilde{J}_x+Z_{zz}\tilde{J}_z.\end{aligned} \tag{5}$$

The procedure just described basically is the derivation of the Greens functions for a layered structure with a delta current source at $y=0$.

This step, however, can become quite tedious, especially when a higher number of layers is involved, as both TM and TE field components depend on both $\tilde{J}_x$ and $\tilde{J}_z$.

II. Spectral Domain Immittance Matrix Approach

An enormous facilitation to calculate the matrix elements $\tilde{Z}_{ij}$ was given by the immittance matrix approach [3], [4]. Its explanation, e.g., in [2] and [4], however, was done in a more illustrative way, and some experience is required to follow this argumentation. Therefore, a direct derivation of this approach will be given here which might be more easy accepted, e.g., when this method is being taught to students.

The field components in the different layers tangential to the $x-z$-plane as given in (4) can be rearranged in the following way:

$$\vec{\tilde{E}}_{\tan} = \begin{pmatrix} \tilde{E}_x \\ \tilde{E}_z \end{pmatrix} = -\frac{1}{\omega\varepsilon}\frac{\partial\tilde{\Phi}}{\partial y}\cdot\begin{pmatrix}\alpha\\ \beta\end{pmatrix} + j\tilde{\Psi}\cdot\begin{pmatrix}-\beta\\ \alpha\end{pmatrix} = \begin{pmatrix}\alpha & -\beta\\ \beta & \alpha\end{pmatrix}\begin{pmatrix}-\frac{1}{\omega\varepsilon}\cdot\frac{\partial\tilde{\Phi}}{\partial y}\\ j\tilde{\Psi}\end{pmatrix}, \quad (6a)$$

$$\vec{\tilde{H}}_{\tan} = \begin{pmatrix} \tilde{H}_x \\ \tilde{H}_z \end{pmatrix} = j\tilde{\Phi}\cdot\begin{pmatrix}\beta\\ -\alpha\end{pmatrix} - \frac{1}{\omega\mu}\frac{\partial\tilde{\Psi}}{\partial y}\cdot\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\alpha & -\beta\\ \beta & \alpha\end{pmatrix}\begin{pmatrix}-\frac{1}{\omega\mu}\cdot\frac{\partial\tilde{\Psi}}{\partial y}\\ -j\tilde{\Phi}\end{pmatrix}. \quad (6b)$$

Looking, for example, at the tangential electric field components of the TM field type, it can be seen that both x -and z-component are proportioned to $-j\frac{1}{\omega\varepsilon}\cdot\frac{\partial\tilde{\Phi}}{\partial y}$, they only differ by a factor α or β.

Similar dependencies exist for the TE-type as well as for both field types of the magnetic field. Therefore, for fixed α and β, the directions of the fields in the $x-z$-plane are fixed, also, as they are drawn, for example, in Fig. 2. From this figure, it can be seen that the field components of TM- and TE-types are orthogonal to each other. This fact is true for *all* combinations of α and β.

Furthermore, a transformation of the principal axis from the $x-y-z$ to the η-y-ξ-system as indicated in Fig. 2 reduces the number of field components from five to three for each type. (Fig. 2 matches with [2] if η is taken as v and ξ as u, respectively).

As can be seen immediately from this figure, the TM-type now has only $E_\eta-$, E_y and $H_\xi-$ components, while the TE-type now is composed of $E_\xi-$, $H_\eta-$, and H_y-field components. Consequently, as the TM-type now has only a H_ξ-component, it is coupled to a J_η surface currents density only, and the TE-type with its H_η-component to J_ξ only. This transformation can be seen, too, in a mathematical way as indicated in the second identities of (6a) and (6b).

Once TE- and TM-fields are *decoupled*, they can be regarded *independently* while satisfying the boundary conditions between the different layers as described in the introduction. As, however, there is only *one* type of field involved in each of these procedures, equivalent transmission line circuits can be used equally to this extent, as it is known from the immittance matrix approach.

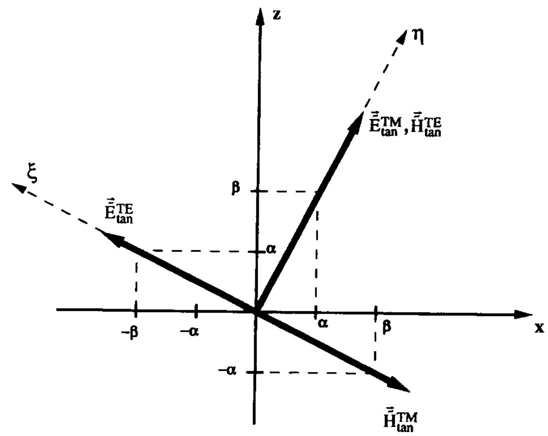

Fig. 2. Coordinate systems and field components parallel to the substrate layers.

Finally, of course, retransformation from the (η, y, ξ)- to the (x, y, z)-coordinate system is done, and the standard Galerkin's procedure is applied to solve the problem.

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REFERENCES

[1] T. Itoh and R. Mittra, "Spectral-domain approach for calculating the dispersion characteristics in microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 496–499, 1973.

[2] T. Itoh, Ed., *Numerical Techniques For Microwave and Millimeter-Wave Passive Structures*. New York: John Wiley, 1989.

[3] T. Itoh, "Spectral domain immittance approach for dispersion characteristics of shielded microstrips with tuning septums," in *Proc. 9th European Microwave Conf.*, 1979, Brighton, pp. 733–736.

[4] ——, "Spectral domain immittance approach for dispersion characteristics of generalized printed transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 733–736, 1980.