

# A Global Finite Difference Time Domain Analysis of a Silicon Nonlinear Transmission Line

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## ABSTRACT

This paper presents a global finite difference time domain (FDTD) analysis of a silicon nonlinear transmission line (NLTL) using optimized varactors. The simulation is based on the FDTD method [1],[2] also including transmission line losses and the drift-diffusion model for the semiconductor devices solved by means of finite differences (FD). The diodes are included in the FDTD scheme as lumped elements. The fall time of 74 ps of a 4 GHz sinewave was compressed to approximately 15 ps at the output of a 20 mm long NLTL with 40 diodes. Comparing the measured output signal to the simulation, a good agreement could be achieved.

## INTRODUCTION

Due to the rapid advances in bandwidth and speed of electronic systems, picosecond pulses need to be generated for signal detection in high-speed sampling circuits. These short transients have successfully been realized with nonlinear transmission lines processed on GaAs in recent years. A NLTL consists of a high impedance coplanar line with distributed diodes along the line.

Due to the voltage dependent capacitance of the diodes, nonlinear propagation effects are introduced. With a shockwave NLTL as described here, the falling edge of a large time-domain signal is steepened. The propagation of the NLTL's and their properties have been investigated since 30 years with different methods, and a few attempts for an analysis and design have been proposed. Many of these approaches, however, underlie several limitations because simplified models for NLTL's are employed. In previous studies [8], circuit simulators are used to characterize the NLTL. The NLTL is approximated by an ideal transmission line loaded with the diodes represented by their large-signal equivalent circuit. Due to the high frequencies which are generated on the line, this simple transmission line model has to be replaced by an electromagnetic model where wave effects can be taken into account. In [7], a lossless FDTD analysis of a NLTL was presented including the diodes by their equivalent circuits. Due to the length of NLTL's and the very broadband spectrum of the generated signal, transmission line losses have an remarkable influence on the propagation properties especially on the steepness of the total fall time. As a consequence, this at-

tenuation has to be considered in an extremely wide frequency range. Therefore in this contribution, the metallization losses are considered in FDTD for the complete frequency range using the surface impedance approach. Furthermore, in contrast to other standard NLTL's, an optimized inhomogeneous doping profile was designed to achieve a faster compression of the falling edge of the output signal. For a better and faster large-signal modelling of the diodes, the device response is extracted from the geometry and doping profile itself applying the finite difference method to the drift-diffusion model as it was previously done in [9] and [10].

### TRANSMISSION LINE LOSSES

For modelling the attenuation on a transmission line, an ultra wide band approximation of metallization losses using the surface impedance approach based on a two-port model for the surfaces is included. A plane conducting sheet with the conductivity  $\sigma$  and the thickness  $t$  is assumed. The electric and magnetic fields on the top and the bottom side of the sheet are described by the following impedance matrix in the frequency domain

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \quad (1)$$

where

$$Z_{11} = Z_{22} = \sqrt{\frac{j\omega\mu}{\sigma}} \coth \sqrt{j\omega\mu\sigma d^2} \quad (2)$$

and

$$Z_{12} = Z_{21} = \sqrt{\frac{j\omega\mu}{\sigma}} / \sinh \sqrt{j\omega\mu\sigma d^2} \quad (3)$$

The impedance matrix transformed into the time domain together with Faraday's equations yield a boundary condition for the magnetic field components surrounding the conducting sheet

[3]. Using a recursive convolution method, the computational effort is small compared to the FDTD calculation of the remaining structure.

### DIODE MODELLING

In the NLTL, each semiconductor device is modeled by the semiconductor transport equations. For arbitrary doping profiles, this approach provides an accurate and efficient model. In the quasistatic case, where the diodes are considered to be small compared to the wavelength, the Poisson equation

$$\Delta\psi = \frac{q}{\epsilon} (n - p - N_D + N_A) \quad (4)$$

can be employed for the potential within the diodes. Together with the charge conservation for electrons and holes

$$\nabla \vec{J}_n - q \frac{\partial n}{\partial t} = qR \quad \text{and} \quad (5)$$

$$\nabla \vec{J}_p + q \frac{\partial p}{\partial t} = -qR \quad (6)$$

and the current densities

$$\vec{J}_n = -qn\mu_n \nabla\psi + qD_n \nabla n \quad \text{and} \quad (7)$$

$$\vec{J}_p = -qn\mu_p \nabla\psi - qD_p \nabla p \quad (8)$$

the system of differential equations yield a solution for the unknown functions  $\psi, n, p, J_n$  and  $J_p$ . The numerical integration is done by means of the FD method. Here, because of the large number of diodes, a one dimensional grid is chosen which is appropriate in this case. The grid is adopted to the geometry of the diode and is independent of the FDTD mesh. For the current densities, the Scharfetter-Gummel integration is applied [5]. The interface between the device simulator and electromagnetic simulator is obtained with Ampere's law

$$\frac{\partial}{\partial t} E|^{n+1/2} \epsilon = \nabla \times H|^{n+1/2} - J^{n+1/2} \quad (9)$$

where the current and voltage at the device and at two grid points are related to each other employing an implicit algorithm.

## NLTL AND RESULTS

A detailed description of the design principles of this NLTL (Fig.1) based on an elevated coplanar line can be found in [6]. The advantage of

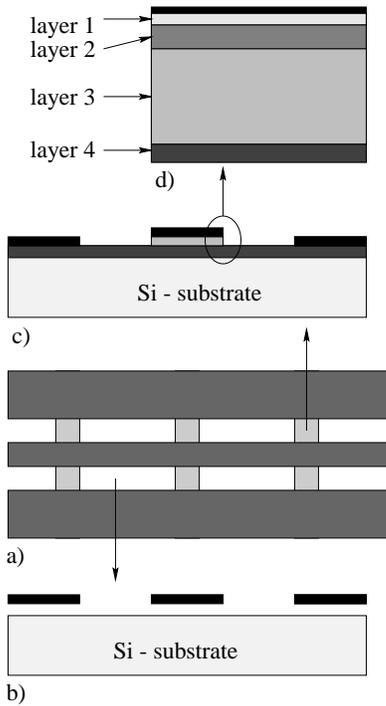


Figure 1: Nonlinear transmission line; layer 1:  $t=35\text{ nm}$ ;  $N_D=10^{16}\text{ cm}^3$ ; layer 2:  $t=80\text{ nm}$ ;  $N_D=10^{17}\text{ cm}^3$ ; layer 3:  $t=550\text{ nm}$ ;  $N_D=2.5 \times 10^{16}\text{ cm}^3$

this kind of coplanar line is the high characteristic impedance of  $106\ \Omega$  and the low attenuation of about  $0.1\text{ dB/mm}$  at  $40\text{ GHz}$ . The attenuation of the passive structure (without diodes) resulting from the FDTD simulation (Fig.2) is compared to the mode matching results [4] for validation. For a fast compression of the input signal, diodes with a non-uniform doping profile are used (Fig.1). This profile provides a higher capacitance swing, yielding a stronger variation of the wave velocity. The capacitance versus the bias voltage is shown in Fig. 3. Both the resistance of the  $n^+$ -layer of about  $12\ \Omega$  and the leakage resistance of about  $700\ \Omega$  are added as

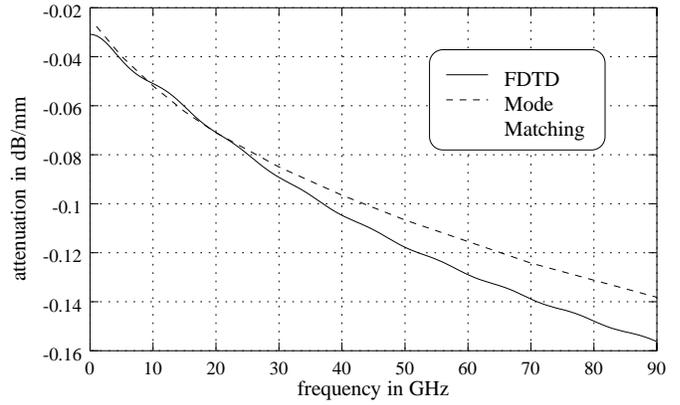


Figure 2: Attenuation of the passive coplanar line (without diodes) on silicon substrate;  $s=40\ \mu\text{m}$ ;  $w=20\ \mu\text{m}$ ;  $t=3\ \mu\text{m}$ ; elevation of the conductors:  $h=3\ \mu\text{m}$

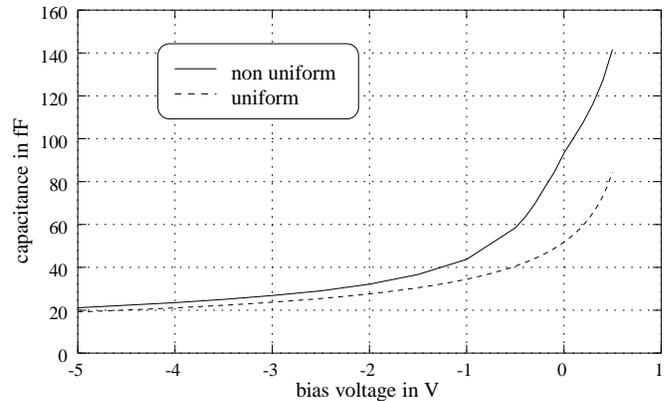


Figure 3: Capacitance variation over bias voltage at  $10\text{ GHz}$ ; uniform doping:  $t=550\text{ nm}$ ;  $N_D=2.5 \times 10^{16}\text{ cm}^3$

lumped elements to the diode circuit as shown in Fig.4. This leakage current through the Schottky barrier results in a high DC resistance and consequently in a high additional attenuation. For a large signal impedance of  $50\ \Omega$ , diodes with a Schottky area of  $100\ \mu\text{m}^2$  are placed in a distance of  $500\ \mu\text{m}$ . The maximal achievable fall time of the output signal could be expressed by  $t_{fall} = 8/(\omega_g * (\Delta C/C_0))$ , where  $\omega_g$  is the cutoff frequency of the diode,  $\Delta C$  is the capacitance swing and  $C_0$  is the capacitance of the linear line section between two diodes. This will

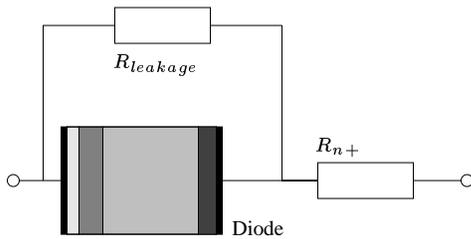


Figure 4: Lumped element included in FDTD

result in an approximately 10 ps steep falling edge. Half of the NLTL structure was simulated using  $30 \times 37 \times 800$  cells, and 40 times a 1D grid with 33 cells for each diode. On a Pentium PC with 500 MHz, about 50 hours are required for 50000 time steps. On the NLTL, an output sig-

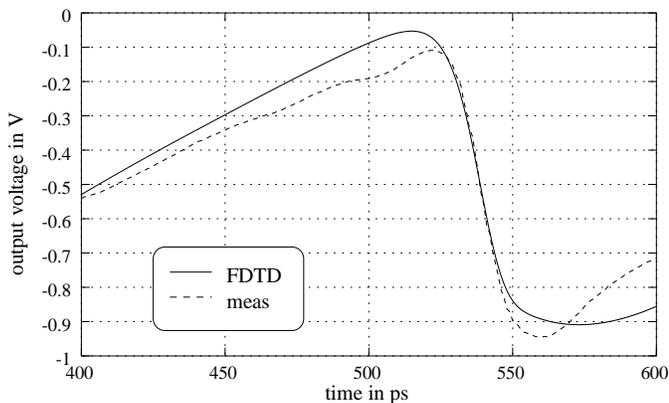


Figure 5: Output pulse of the NLTL

nal of 0.8 V amplitude and a fall time of 15 ps was observed which is confirmed by the FDTD simulation shown in Fig.5.

## CONCLUSION

A global FDTD method for an efficient analysis of NLTL is presented. The transmission line losses are included over a wide frequency range, and the non-uniform doping profile of the diodes can be considered by applying the FD method to the semiconductor transport equations. This global model also enables an optimization of the NLTL's and diodes. As a consequence, fewer

technology runs are required.

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