On the Relation between a Negative Refractive Index Transmission Line and Chebyshev Filters

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***Abstract*—The relation between composite right/left handed transmission lines (CRLH TL) and lumped element Chebyshev filters is discussed in this paper. A CRLH TL in the balanced case can be regarded as the central part of a low-ripple high-order Chebyshev band-pass filter (BPF). The balanced case of the CRLH TL is automatically satisfied in the mapping from a prototype Chebyshev low-pass filter (LPF) to a BPF. Moreover, once the ending sections are take into consideration, both a better impedance matching is achieved as well as a reduction of the large ripples close to the cut-off frequencies for a finite length CRLH TL. Therefore, a CRLH TL in the balanced case may be designed from a Chebyshev filter exhibiting an improved performance.**

I. INTRODUCTION

In the last years, metamaterial structures [1] have found a wide interest. A homogeneous negative index transmission line (TL) or left-handed (LH) transmission line does not exist in nature. It has to be approached by an artificial structure which is usually constructed from a series of discontinuous sections operating in a restricted frequency range. A typical realization is found in a quasi-lumped transmission line with elementary cells consisting of a series capacitor and a shunt inductor [2]. As in practice, the normal shunt capacitance and series inductance cannot be avoided, the concept of the composite right/left-handed transmission line (CRLH TL) was developed, and a number of novel applications have been demonstrated [2 - 6].

Fig. 1 shows the equivalent circuit of such a transmission line (connected to a source and a load). Without the right-handed elements (e.g. L_1 and C_2 or L_j and C_k), such a circuit looks like a highpass filter, and the complete CRLH TL like a band-pass filter (of typically high order).

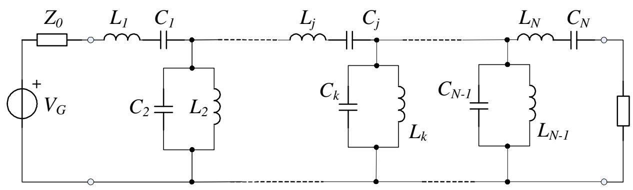

Fig. 1 CRLH TL's LC circuit model

A conventional filter, on the other hand, is generally not a uniform structure. Looking, however, at the filter coefficients g_i of a Chebyshev filter, it can be seen that such a filter with high order exhibits a highly periodic central section; in the limiting case, it can be considered as a periodic structure with matching sections at both ends (see Fig. 2).

In this paper, we therefore compare the design of CRLH TLs and of Chebyshev filters of high order. It is proved that a CRLH TL can be considered as a part of a Chebyshev filter. Thus, if we design a CRLH TL based on classical filter theory, a better performance of a finite length section can be achieved. To this end, we firstly summarize the characteristic formulas of a CRLH TL. Secondly, we analyse the design formulas for a Chebyshev band-pass filter with a similar characteristic and prove that the central section of a Chebyshev band-pass filter is identical to a CRLH TL. By attaching the ending sections of the filter, a finite length CRLH TL with better performance is achieved.

II. CRLH TL IN THE BALANCED CASE

The equivalent circuit model of a CRLH TL is a cascaded network as shown in Fig. 1 which consists of series inductance (L) and capacitance (C) resonators alternating with shunt LC resonators. Both source and load resistance are Z_0. Since the arrangement is a periodic structure, those LC components are

$$\begin{cases} L_j = L_R, \quad C_j = C_R, & \text{for series resonators} \\ L_k = L_L, \quad C_k = C_L, & \text{for shunt resonators} \end{cases} \tag{1}$$

where the subscript L and R denotes LH and right handed (RH), respectively. In a balanced CRLH TL, there is no frequency band gap between the RH and LH region, and it provides better impedance matching over a broader frequency range as well. The balanced condition of the CRLH TL is [3, 4]

$$L_L C_R = L_R C_L \ . \tag{2}$$

The transition angular frequency between LH and RH region is

$$\omega_0 = \omega_{Se} = \omega_{Sh} \tag{3}$$

where $\omega_{Se} = \dfrac{1}{\sqrt{L_R C_L}}$ and $\omega_{Sh} = \dfrac{1}{\sqrt{L_L C_R}}$ are the resonant frequencies of the series LC circuit and the shunt LC circuit, respectively. The equivalent characteristic impedance Z_E of a CRLH TL in the balanced case is

$$Z_E = Z_L = Z_R \tag{4}$$

where $Z_L = \sqrt{\frac{L_L}{C_L}}$ and $Z_R = \sqrt{\frac{L_R}{C_R}}$ are the pure LH and RH characteristic impedances which are frequency independent with the homogenous TL approach. Assuming an infinitely long CRLH TL, the lower and higher cut-off frequencies are

$$\begin{cases} \omega_{cL} = \omega_R\left(\sqrt{1+\frac{\omega_L}{\omega_R}}-1\right) \\ \omega_{cR} = \omega_R\left(\sqrt{1+\frac{\omega_L}{\omega_R}}+1\right) \end{cases} \tag{5}$$

where $\omega_L = \frac{1}{\sqrt{L_L C_L}}$ and $\omega_R = \frac{1}{\sqrt{L_R C_R}}$ are the resonant frequencies of the LH and RH LC circuit, respectively. From Eq. (2), (3) and (5), the relation between transition frequency and cut-off frequencies is

$$\omega_0^2 = \omega_{cL}\omega_{cR} = \omega_L\omega_R = \frac{1}{\sqrt{L_L L_R C_L C_R}} \tag{6}$$

which is similar to that of a BPF. While $\omega_{cL} < \omega < \omega_0$, the CRLH TL is dominantly LH, and while $\omega_0 < \omega < \omega_{cR}$, it is dominantly RH.

Considering the degree of freedom in the CRLH TL design with periodical elements, there are four independent parameters, namely L_L, C_L, L_R, and C_R. When the balanced condition is applied, there are only three independent parameters left. Once two cut-off frequencies (as well as the transition frequency) and the characteristic impedance Z_E (matching to the system impedance Z_0) are fixed, a unique CRLH TL configuration is determined based on Eq. (2) to (6) as

$$\begin{cases} \omega_0 L_R = \frac{1}{\omega_0 C_L} = \frac{\omega_0}{\omega_{cR}-\omega_{cL}} 2Z_0 \\ \omega_0 C_R = \frac{1}{\omega_0 L_L} = \frac{\omega_0}{\omega_{cR}-\omega_{cL}} \frac{2}{Z_0} \end{cases} \tag{7}$$

When a mismatch between the characteristic impedance of a balanced CRLH TL and the system impedance occurs, matching circuits have to be implemented between the CRLH TL and the source or load. The matching circuit can be a tapered microstrip line, which is quite long and increases the insertion loss.

III. CHEBYSHEV BAND-PASS FILTER

An N[th] order band-pass filter (BPF) in principle has the same LC circuit model as that of the CRLH TL in Fig. 1 (in this section, odd order Chebyshev filters are considered; even order filters are shortly discussed in section IV). The BPF design is usually achieved from the low-pass to band-pass transformation, in which a low-pass prototype filter is applied. The mapping formulas can be found in [7] with

$$\begin{cases} \omega_0 L_j = \frac{1}{\omega_0 C_j} = \frac{\omega_0}{\omega_2-\omega_1} g_j Z_0 & \text{for series resonators} \\ \omega_0 C_k = \frac{1}{\omega_0 L_k} = \frac{\omega_0}{\omega_2-\omega_1}\frac{g_k}{Z_0} & \text{for shunt resonators} \end{cases} \tag{8}$$

where g_i is the i^{th} element value (either the inductance or the capacitance) in a prototype LPF, ω_1 and ω_2 are the lower and higher cut-off frequencies, respectively, and $\omega_0 = \sqrt{\omega_1\omega_2}$ is the central frequency of the BPF. Z_0 is the system impedance. Once these parameters are fixed, the BPF is uniquely determined. From Eq. (8), it can be obtained

$$L_j C_j = L_k C_k = \frac{1}{\omega_0^2} \tag{9}$$

Eq. (9) is equivalent to Eq. (2) and (3), which means that the balanced condition of a CRLH TL always holds in BPF design. Since the mapping formula of Eq. (8) is a generic formula, it may be applied to other kinds of prototype LPF as well. Thus the balanced case of a CRLH TL is automatically realized in BPFs from ***any kind*** of prototype LPF with series and shunt LC resonators that are built from the mapping formula Eq. (8). Butterworth, Gaussian, or Chebyshev BPFs with any passband ripple constructed from Eq. (8) will satisfy the balanced condition of a CRLH TL. In most prototype LPFs, the element values g_i usually vary in a certain range and lead to a non-periodic structure. The central section of a high order Chebyshev filter, however, has a periodical structure and is very close to a CRLH TL. Fig. 2 shows an example of a 21[st] order Chebyshev prototype LPF with different pass-band ripples. For elements not close to either end, the element values are periodical. It should be noted that always two adjacent filter elements form one equivalent TL cell, thus the central part of a Chebyshev is really a periodic structure, independently of the ripple.

A. Low Pass-Band Ripple

As shown in Fig. 2, the lower the pass-band ripple, the smoother are the element values. In the limiting case, the values of the central elements are close to $g_i \approx 2$. Moreover, the higher the filter order, the better is the approach to a periodic structure. When the pass-band ripple in a Chebyshev

BPF is low enough and its order high enough, it can be proved theoretically or shown numerically that the filter element values are $g_i=2$ for all elements that are not close to either filter end.

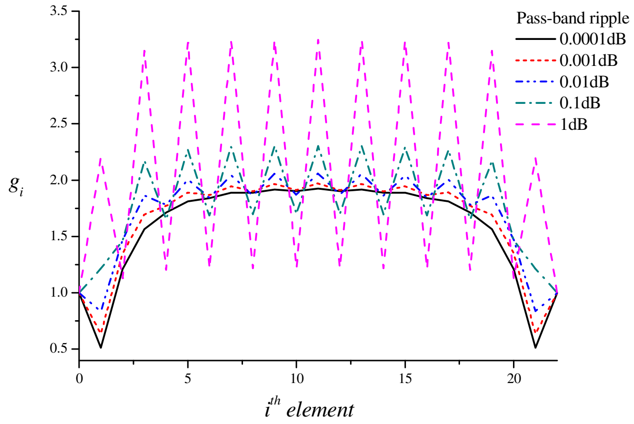

Fig. 2 Element values of 21st order Chebyshev prototype LPFs

When the element value is 2, Eq. (7) and Eq. (8) will have the same form. Thus, when the Chebyshev BPF and the CRLH TL have the same cut-off frequencies ($\omega_1=\omega_{cL}$, $\omega_2=\omega_{cH}$) and system impedance Z_0, they will own the same LC circuit determined by Eq. (7) and Eq. (8), respectively. This implies that once three parameters (two cut-off frequencies and the matching impedance) are fixed, the corresponding CRLH TL in the balanced case and the central part of the corresponding high order Chebyshev BPF with low pass-band ripple have identical LC configurations.

B. High Pass-band Ripple

With higher pass-band ripple in a Chebyshev prototype LPF, the element values oscillate instead of approaching a constant $g_i \approx 2$ (Fig. 2). It can also be proved that the values of central elements switch between two fixed values. For those elements, the element value of series and shunt resonators keep constant as g_{se} and g_{sh} alternatively, and $g_{se}\cdot g_{sh}=4$ always holds. Thus, a high-ripple Chebyshev BPF is a periodic configuration as well. In fact, it is also a kind of the CRLH TL in a balanced case, as will be discussed in section IV.

Once two cut-off frequencies and the system impedance are given, a uniform CRLH TL in the balanced case and a high order Chebyshev BPF with low pass-band ripple can be uniquely implemented with LC circuit, respectively. The CRLH TL and the central part of the Chebyshev BPF own the same periodical LC structures. In other words, **a uniform CRLH TL in the balanced case can be considered a part of a high-order low-ripple Chebyshev BPF with the same cut-off frequencies and system impedance.**

IV. Impedance Matching and Pass-Band Ripple

The image impedance has more general meanings than its definition with respect to the characteristic impedance of a uniform TL [7]. From the image impedance method, the characteristic impedance of a balanced CRLH TL is

$$Z_E = Z_R\sqrt{1-\frac{\varepsilon^2}{4}\frac{\omega_L}{\omega_R}} \tag{10}$$

where $\varepsilon=\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}$. Eq. (10) shows that the characteristic impedance Z_E is frequency dependent, as shown in Fig. 3. Thus, only around the transition frequency, Eq. (4) can be obtained. This indicates that impedance matching circuits should be applied to finite length CRLH TLs for better performance.

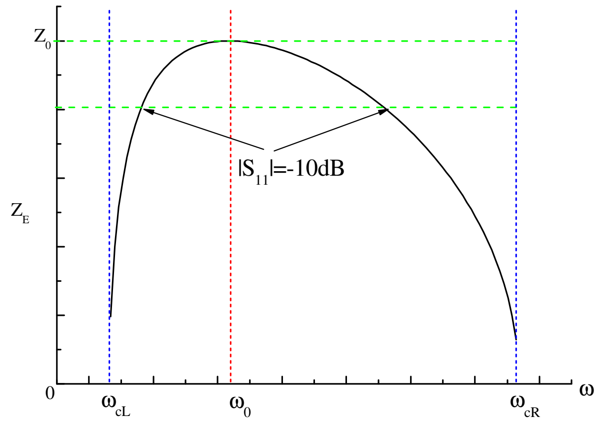

Fig. 3 Characteristic impedance of CRLH TL

When the pass-band ripple in a Chebyshev prototype LPF is high, the values of central elements are alternatively $g_j=2\alpha$ and $g_k=\dfrac{2}{\alpha}$ for series and shunt element, and α is a positive constant. If Z_0 in Eq. (8) is replace by αZ_0, it is still a well designed CRLH TL. Therefore, the pass-band ripple corresponds to the difference between the applied and designed system impedance of Chebyshev BPF. The greater the difference is, the higher is the pass-band ripple. Based on the element value analysis of a high-order Chebyshev prototype LPF, the relation between the pass-band ripple and the impedance difference is

$$\alpha^s=\sqrt{\frac{g_{se}}{g_{sh}}}=\coth\left(\frac{1}{2}\sinh^{-1}\left(10^{\frac{L_{Ar}}{10}}-1\right)^{-\frac{1}{2}}\right) \tag{11}$$

where L_{Ar} is the pass-band ripple in dB, $\alpha = \frac{Z_0^{\text{Designed}}}{Z_0^{\text{Applied}}}$ the ratio of the applied and designed impedances, and the sign function $s = \begin{cases} +1 & \forall Z_0^{\text{Desinged}} > Z_0^{\text{Applied}} \\ -1 & \forall Z_0^{\text{Desinged}} < Z_0^{\text{Applied}} \end{cases}$. When α is 1, there is no ripple in pass-band. Once α is given, the pass-band ripple can be obtained easily. For example, if the applied system impedance is 50Ω and the characteristic impedance of a CRLH TL 70Ω, the corresponding pass-band ripple in a BPF will be $L_{Ar} = 0.4827dB \approx 0.5\text{dB}$.

When the applied and designed impedances are different, matching circuits are necessary. Even in the matched situation, it is required to improve the overall pass-band performance as well. Besides the tapered line matching method, another method is to utilize the classical filter design techniques. Since a CRLH TL is proved to be equivalent to the central part of a Chebyshev BPF, the impedance matching sections between the CRLH TL and the source or load can be constructed from the corresponding BFP design. This procedure is equivalent to the design of an entire filter and automatically results in good performance. Broadband impedance matching is realized easily, while the unwanted high ripples close to the cut-off frequencies are reduced and distributed to the whole pass-band smoothly. It is a classical and stable method, which provides much better performance in either the LH or RH region.

In the situation that even order Chebyshev filters are concerned, the load impedance, which is only dependent on the pass-band ripple and independent on the filter order, is always different from the source impedance. However, when the pass-band ripple is small, the load impedance is very close to the source impedance. Therefore, there is not much difference between filter element values of even order and odd order, when the pass-band ripple is low and the order is high. On the other hand, when the pass-band ripple is high, the difference between even order and odd filters is located only in several elements near the load, which can be understand as an impedance matching circuit. With respective to even order Chebyshev filters, the analysis in this paper is also suitable.

V. CONCLUSIONS

The characteristics of CRLH TLs (a kind of negative refractive index TL) and high order Chebyshev BPFs are analysed, and synthesis formulas based on matching impedance and cut-off frequencies are shown. From the analysis of element values in Chebyshev prototype LPF, the relation between CRLH TL and Chebyshev BPF is revealed. It is proved that a CRLH TL - in the balanced case - is the central part of a high-order low pass-band ripple Chebyshev BPF with identical matching impedance and cut-off frequencies.

The meaning of pass-band ripple in a Chebyshev BPF, which has no obvious counterpart in a CRLH TL, corresponds to the mismatch between the characteristic impedance of the CRLH TL and the system impedance. The formula to compute pass-band ripple from impedance mismatch is presented. In addition, impedance matching circuits in CRLH TL applications can be design based on classical filter theory to achieve much better performance.

The other way round, a CRLH TL can be designed from a Chebyshev BPF. By allowing a reasonable pass-band ripple, there is more design freedom, and with respect to impedance matching, the design from classical filter theory can achieve smooth broadband responses.

First examples for such an approach have already been presented in [6] and [8] where sections of a CRLH TL have been realized as band-pass filters to design antennas with backfire-to-endfire scanning.

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