New pre-estimation Algorithm for FMCW Radar Systems using the Matrix Pencil Method

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***Abstract*—This paper shows a technique to enhance the resolution of a frequency modulated continuous wave (FMCW) radar system. The range resolution of an FMCW radar system is limited by the bandwidth of the transmitted signal. By using high resolution methods such as the Matrix Pencil Method (MPM) it is possible to enhance the resolution. In this paper a new method to obtain a better resolution for FMCW radar systems is used. This new method is based on the MPM and is enhanced to require less computing power. To evaluate this new technique, simulations and measurements are used. The result shows that this new method is able to improve the performance of FMCW radar systems.**

***Keywords*—Radar, FMCW, High Resolution, Matrix Pencil Method, Signal Processing**

I. Introduction

An important requirement in many radar applications is a high range resolution. If more than one object is in the detection range, the radar system should be able to separate among different objects. To improve the separability a higher resolution is necessary. Usually the resolution of a radar system is limited by the bandwidth of the transmitted signal. To enhance the resolution without changing the bandwidth, high resolution algorithms can be used. In literature different high resolution algorithms are mentioned, such as the Matrix Pencil Method (MPM) [1], [2], Multiple Signal Classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [3]. One limitation of these high resolution algorithms is the computational time. To reduce the computational cost different algorithms of eigenstructure decompositions have been suggested [3], [4]. Another idea is to use the high resolution processing only if the objects are relevant [5].

In this paper an adaptive algorithm is used for FMCW radar systems and the high resolution processing is only used for relevant objects. The proposed method uses the high resolution algorithm only if the nearest object can not be determined exactly. At first in the 2. section the signal processing of an FMCW radar is shown. Section 3 describes the MPM and in section 4 the new algorithm is explained. Section 5 shows the performance in simulations and measurements and finally in section 6 the conclusion of this work is given.

II. FMCW Radar

An FMCW radar system is a frequently used modulation technique in automotive systems. This section gives an overview of the FMCW radar basics.

A. Modulation

To measure the distance and the relative velocity of an obstacle an FMCW radar system generates a frequency-modulated continuous wave. The transmitted signal is reflected at a target and received by the radar system. To obtain the beat frequency f_b the receiving signal is mixed with the transmitted signal. The beat frequency contains the distance and the relative velocity of a detected target. In this paper, only static scenarios are investigated which means that the relative velocity is zero. In this case the beat frequency is expressed by (1)

$$f_\mathrm{b} = \frac{2\Delta f R}{c_0 T} \tag{1}$$

with the speed of light c_0, the target range R and the modulation parameters bandwidth Δf and duration T [6]. The range resolution is given by (2) [6]

$$\Delta R = \frac{c}{2\Delta f}. \tag{2}$$

As seen in (2) the range resolution, and therefore the range separability, is limited by the bandwidth.

B. Signal Processing

Usually the received time signal is multiplied with a window function and transformed to the frequency domain with a fast Fourier transform (FFT). A threshold calculation and a peak finding algorithm calculate the frequency of a certain object. The radar system can not separate objects, which are situated closely together. To separate these objects a larger bandwidth or a high resolution algorithm needs to be applied. In this paper the high resolution algorithm MPM is used to enhance the resolution.

III. Matrix Pencil Method

The Matrix Pencil Method is a high resolution method. With this method it is possible to estimate the parameters of a signal which can be expressed as [1], [2]

$$y(t) = x(t) + n(t) = y(kT_\mathrm{s}) \approx \sum_{i=1}^{M} R_\mathrm{i} z_\mathrm{i}^k + n(kT_\mathrm{s}) \tag{3}$$

with $k = 0, 1, ..., N-1$ and $z_\mathrm{i} = e^{s_\mathrm{i} T_\mathrm{s}}$ where

$y(t)$ = time signal,
$x(t)$ = ideal signal,

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$n(t)$ = noise,
R_i = residues or complex amplitude of the i. reflection,
s_i = $-\alpha_\mathrm{i} + j\omega_\mathrm{i}$,
α_i = damping factor of the i. reflection,
ω_i = angular frequency of the i. reflection,
M = model order,
T_s = sampling period.

The aim of this method is to estimate the parameters M, R_i, z_i. The following section shows the algorithm. At first the noise-containing data is sorted in a matrix $Y \in \mathbb{C}^{(N-L)\times(L+1)}$

$$Y = \begin{bmatrix} y(0) & y(1) & ... & y(L) \\ y(1) & y(2) & ... & y(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(N-L-1) & y(N-L) & ... & y(N-1) \end{bmatrix} \quad (4)$$

where N is the number of samples and L is the Pencil Parameter, which has a value between $N/2$ and $N/3$ for efficient noise filtering [1], [2]. This method can be used for an FMCW radar system by sorting the sampled time data in the matrix Y.

Afterwards a singular-value decomposition (SVD) of Y is implemented

$$Y = UDV^H. \quad (5)$$

In (5) U is an unitary matrix with the eigenvectors of YY^H and V is an unitary matrix with the eigenvectors of Y^HY. D is a diagonal matrix including the singular values of the matrix Y. The model order M is either specified or estimated with the singular values of Y. The highest singular values are related to the signal and the quantity of these values equals the model order, the other singular values are related to the noise [2].

To eliminate the noise in [2] a simplified model of the matrix is used

$$Y_1 = U_\mathrm{s} D_\mathrm{s} V_{1\mathrm{s}}^H \quad (6)$$

$$Y_2 = U_\mathrm{s} D_\mathrm{s} V_{2\mathrm{s}}^H \quad (7)$$

where D_s contains the first M columns of D. $V_{1\mathrm{s}}$ contains the first M columns of V with the last row deleted and $V_{2\mathrm{s}}$ contains the first M columns of V without the first row. The matrix U_s contains the first M rows of the matrix U [2], [7].

The parameters z_i correspond to the eigenvalues λ of the following matrix [2]

$$\{Y_2 - \lambda Y_1\}_{L\times M} \Rightarrow \{Y_1^+ Y_2 - \lambda I\}_{M\times M}. \quad (8)$$

With the Parameters M and z_i the parameters R_i are calculated by solving the following equation.

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & ... & 1 \\ z_1 & z_2 & ... & z_\mathrm{M} \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{(N-1)} & z_2^{(N-1)} & ... & z_\mathrm{M}^{(N-1)} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_\mathrm{M} \end{bmatrix} \quad (9)$$

After solving (9) all parameters of the signal model (3) are estimated and for each reflection i a complex frequency and amplitude are given.

IV. New high Resolution Algorithm

To enhance the resolution and separability of a radar system a higher bandwidth or a high resolution algorithm is needed. In this paper a new high resolution algorithm with an improved computational efficiency is presented. This new high resolution method is based on the MPM. Similar to Hyun [5] the MPM algorithm is only used for relevant signals. In figure 1 the flowchart of the proposed method is shown.

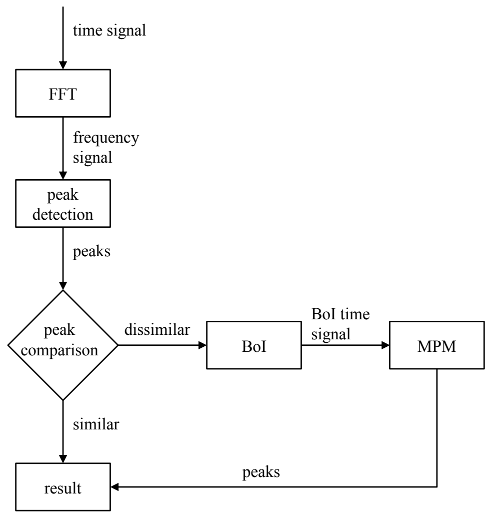


Fig. 1. Flowchart of the new algorithm

The first steps of the new method are similar to a normal FMCW radar signal processing. In addition the detected peaks are investigated after peak detection. To reduce the computational cost only the peak of the nearest target is considered. In most automotive radar applications only the nearest target is relevant and critical regarding the accuracy of the distance measurement. A small distance error for farther objects is tolerable.

The peak of the nearest object is analyzed. Either it is like the expected peak of one target or it consists of superposition of more targets. The expected peak in the frequency domain can be calculated based on the window function. The time signal is multiplied with a window function before using the FFT. This equals a convolution of the signal and the window function in the frequency domain. Typically a signal containing a single frequency equates to a Dirac impulse in the frequency domain. Hence, the expected peak of one target is equal to the frequency domain of the window function. If a signal consists of more than one frequency the full width at half maximum (FWHM) of the peak is different from the expected peak. A measured target always has more than one reflection point and as a result a peak always exists of more than one received frequency that can not be separated. A measured FMCW radar signal is shown in figure 2.

Figure 2 shows a measured signal of a vehicle front at a distance of 1.05 meter. The black dashed line shows the expected peak. By comparing the signal with the expected peak

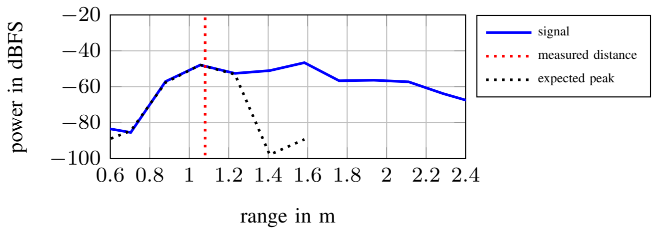


Fig. 2. Measured signal of a vehicle front

it can be seen, that the signal is a result of superposition of more targets. Further it is seen, that the slope of the measured signal peak is similar to the expected slope. Hence the detected distance without using a high resolution algorithm corresponds to the real distance. This behavior is the basis for the new algorithm.

The new proposed method is a pre-estimation algorithm, which compares only the slope of the first signal peak. With this new method it is possible to reduce the computational cost and prevent using the MPM unnecessarily. For example the distance of the nearest target in figure 2 can be calculated without any further high resolution algorithm. The new algorithm prevent using the MPM in this case, because the signal slope is similar to the expected slope. Otherwise if a signal slope is unequal to the expected slope the high resolution method MPM is used to calculate the distance of the first target exactly. To reduce the computational cost a band of interest (BoI) like in [3] is defined as a spectral range related to this peak. Afterwards using the MPM it is possible to separate the different frequencies of the signal.

The MPM algorithm approximates the data by a sum of complex exponentials [2]. Since the measured data is not complex, the signal in the frequency domain is mirrored and the model order has to be duplicated.

V. Evaluation with Simulations and Measurements

To evaluate the new method simulations and measurements are used.

A. Simulations

First a simulation with two input signals is investigated. The input signal consists of two sinus signals A and B with two different frequencies and an added random noise data. This input time signal is multiplied with a Kaiser window ($\beta = 5$) and the frequency domain is plotted in figure 3.

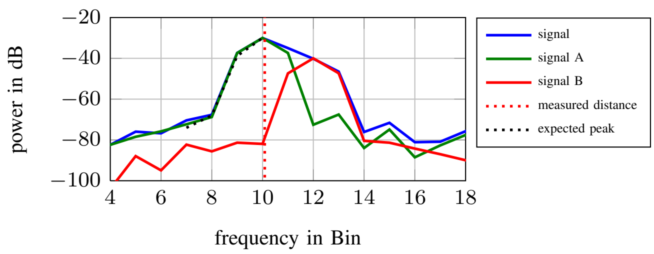


Fig. 3. Simulation of a signal with two different frequencies

Figure 3 shows a signal with two different frequencies and amplitudes. The blue line is the input signal and the dashed black line shows the ideal signal sequence. Signal A with the smaller frequency has a 10 dB higher amplitude than signal B. The distance between these frequencies is not big enough to separate the two frequencies. Since signal A has a much higher amplitude a normal peak detection algorithm can estimate the frequency of this signal accurately. The new method indicates the slope of the peak as the expected slope and no further investigations are necessary.

If the amplitude of signal A is equal or smaller than the amplitude of signal B the algorithm indicates an superposition and the high resolution method MPM is used. The result of the MPM is represented as black points in figure 4. It can be seen, that the MPM algorithm estimates the different input frequencies accurately. If the estimated model order of the MPM is too high, further frequencies are estimated. In figure 4 it can be seen, that these further frequencies have a low power and can be eliminated with a threshold calculation.

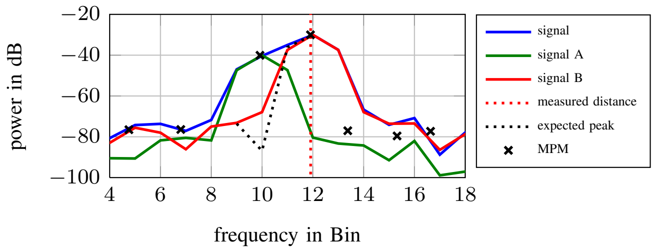


Fig. 4. Simulation of a signal with two different frequencies

In the following section a more complex simulation is shown. Two additional sinus signals are added and presented in figure 5. In addition the expected peak and the result of the MPM is shown.

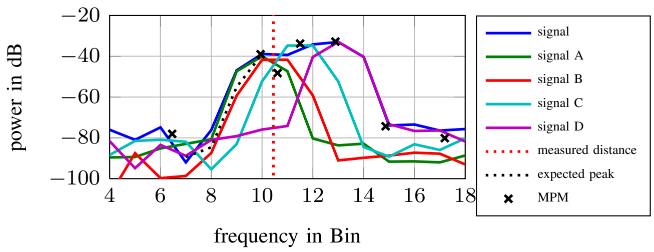


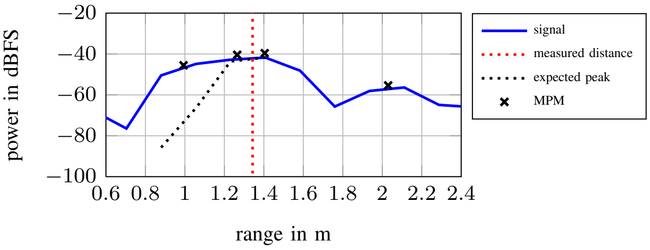
Fig. 5. Simulation of a signal with four different frequencies

Like in figure 4 the slope of the peak is unequal to the expected slope and the MPM is used. Hence, the different frequencies are estimated much better than with normal peak detection algorithms.

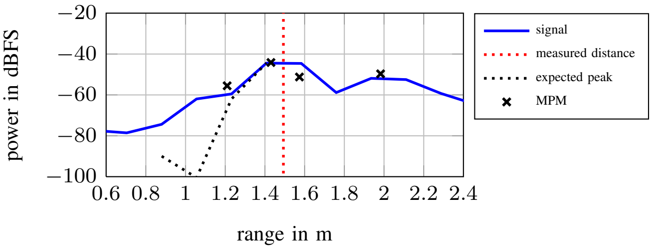
B. Measurements

Measurements of real objects show the quality of the proposed method. In scenario one the front of a car is measured and in scenario two a wooden pole in front of a metal pole is investigated. For these measurements an FMCW radar system with a bandwidth of 850 MHz is used, which results in a separation capability of 17.6 cm according to (2).

1) Car: The measured signal of the front of the car is shown in figure 6.



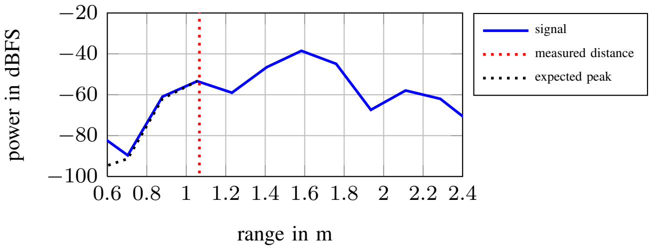
(a) Car at a distance of 1 m



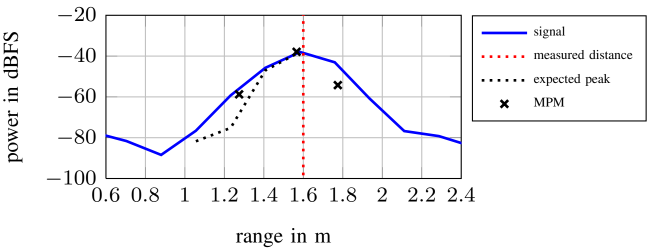
(b) Car at a distance of 1.2 m

Fig. 6. Front of a car

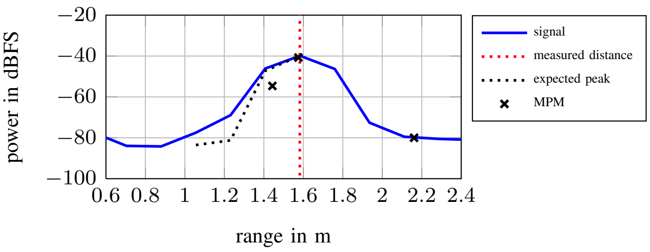
2) Wooden pole in front of a metal pole: The measured signal of the second scenario is shown in figure 7.



(a) 57 cm gap, wooden pole at a distance of 1 m



(b) 27 cm gap, wooden pole at a distance of 1.3 m



(c) 17 cm gap, wooden pole at a distance of 1.4 m

Fig. 7. Wooden pole in front of a metal pole

The wooden pole has a much lower radar cross-section (RCS) than the metal pole, which means that the received power of the wooden pole is much lower than the received power of the metal pole. This scenario can be compared with the simulation shown in figure 4. In figure 7(a) the distance between the wooden pole and the metal pole is big enough to separate these objects. The peak slope is like the expected slope and the MPM is not used to reduce the computational cost.

3) Result: The following table I shows the distance error for the particular measurements. The normal peak finding algorithm is compared to the high resolution method.

TABLE I. DISTANCE ERROR

Scenario	1: 100 cm	1: 120 cm	2: 57 cm	2: 27 cm	2: 17 cm
Standard	34.3 cm	29.3 cm	6.7 cm	30.0 cm	18.2 cm
New	-0.7 cm	1.0 cm	6.7 cm	-2.5 cm	4.3 cm

With the new method the distance error is reduced for both scenarios. Additionally the computational cost is reduced in comparison with the use of the MPM algorithm without any pre-estimation.

VI. Conclusion

In this paper a new method is presented to enhance the resolution of an FMCW radar system. This method is based on the Matrix Pencil method. A pre-estimation algorithm is developed which compares the received signal with an ideally expected signal. To reduce the computational costs a high resolution method, in this paper the Matrix Pencil method, is only used if it is necessary. The proposed algorithm was evaluated with simulations and measurements. A better resolution was shown and the high resolution processing was only used, if the measured range deviates significantly.

References

[1] Y. Hua, and T. Sarkar, *Matrix pencil method for estimating parameters of exponentially damped/undamped sinusoids in noise*, IEEE Transactions on Acoustics, Speech and Signal Processing, vol 38, no. 5, pp. 814-824, May 1990.

[2] T. Sarkar, and O. Pereira, *Using the matrix pencil method to estimate the parameters of a sum of complex exponentials*, IEEE Antennas and Propagation Magazine, vol. 37, no. 1, pp. 48-55, February 1995.

[3] M. Testar, and R. Stirling-Gallacher, *New super-resolution ranging technique for FMCW Radar systems*, Millimetre Wave and Terahertz Sensors and Technology IV, Proc. of SPIE, vol. 8188, October 2011.

[4] N. Yilmazer, J. Koh, and T. Sarkar, *Utilization of a unitary transform for efficient computation in the matrix pencil method to find the direction of arrival*, IEEE Transactions on Antennas and Propagation, vol. 54, no. 1, pp. 175-181, January 2006.

[5] E. Hyun, S.-D. Kim, C. Park, and J.-H. Lee, *Automotive fmcw radar with adaptive range resolution*, in Second International Conference on Future Generation Communication and Networking Symposia, pp. 130-133, December 2008.

[6] G. M. Brooker, *Understanding Millimetre Wave FMCW Radars*, in First International Conference on Sensing Technology, New Zealand, pp. 152-157, November 2005.

[7] M. Pohlmann, *Stationaere Detektion von Objekten in Baumaterialien mittels radarbasierter Verfahren*, 1. Ed., Verl. Dr. Hut, Munich, March 2012.