High-Resolution Parameter Estimation for Chirp-Sequence Radar Considering Hardware Impairments

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***Abstract*—Parametric signal processing is considered for radar measurements to overcome the Rayleigh resolution limit, which is denoted as high resolution. A model describing the measurement data in terms of the radar system and the target parameters is required. A chirp-sequence radar with a stretch processing receiver architecture is considered and an according system model is derived. Based on the maximum-likelihood method and the data model, a parameter estimator is derived to infer the target parameters range and radar cross section from the measurements. The derived radar system model is verified by application of the estimator to real measurement data.**

I. Introduction

Commonly, signal processing techniques as e.g. fast Fourier transform (FFT) based methods are employed in radar signal processing for target parameter inference as e.g. range. Resolution of this methods is limited by the bandwidth of the radar in the respective dimension, the so called Rayleigh resolution limit. Here resolution is denoted as the capability of separating two targets in a certain dimension. Parametric or model based signal processing techniques to infer target parameters can overcome this resolution limit, why such methods are denoted as high resolution parameter estimation (HRPE). A parametric model is fitted to the measured data and therefore a much higher resolution can be achieved. Basically, this resolution is limited by the available signal to noise ratio (SNR) and the model accuracy.

Parametric signal processing consists of two main components, which are the parametric model and an estimator. The parametric model f describes the mapping of the parameters $\boldsymbol{\theta}$, comprising the parameters of interest and additional nuisance parameters, on the measurements y.

$$f : \boldsymbol{\theta} \rightarrow y \tag{1}$$

Model f describes the process of generating sampled data by the measurement system, hence it is called measurement data model. Inferring parameter values from the measurements gives an inverse problem.

$$f^{-1} : y \rightarrow \boldsymbol{\theta} \tag{2}$$

The inverse problem is solved by the estimator, which deduces estimates of the model parameters [1].

The measurement data model consists of two separate models. First, a model describing the propagation from transmitter (Tx) to receiver (Rx) and which comprises the parameters of interest. This model is denoted as the propagation or channel model. Second, a model of the radar system itself, which describes the system architecture and signal distortions due to hardware impairments. This model is denoted as radar system model. In this contribution, a general system model is derived for chirp sequence (CS) radars with stretch processor architecture, which will be validated for the HRPE application.

The remaining document is structured as follows: Section II presents the considered CS radar architecture and derives the radar system model. The data model and a maximum-likelihood (ML) based parameter estimator are derived in Section III. In Section IV, the radar system model is validated using measurements in conjunction with the parameter estimator. Section V concludes the paper.

Mathematical notation is as follows: scalars are italic lower-case letters, vectors in column format are written as bold faced lower-case letters and matrices correspond to bold faced capitals. The matrix operations $(.)^T$ and $(.)^H$ are defined as the transpose and conjugate transpose of a matrix, respectively.

II. Radar System Model

Solving the inverse problem necessitates a proper model of the measurement data, which among others consists of a model of the measurement system itself. The radar system under consideration is a CS radar, which transmits a periodic linear frequency modulated (LFM) (chirp signal). The instantaneous frequency is

$$f(t) = \frac{B_C}{T_C} \cdot t + f_c \qquad -\frac{T_C}{2} \leq t \leq \frac{T_C}{2} \tag{3}$$

with carrier frequency f_c, chirp duration T_C and bandwidth B_C. The signal phase can be calculated from the instantaneous frequency by integration

$$\phi(t) = 2\pi \int_{-T_C/2}^{t} f(t')dt' = 2\pi \left(f_c t + \frac{B_C}{2T_C} t^2 \right) - \phi_0 \tag{4}$$

with ϕ_0 the phase offset. At the receiver, pulse compression and down conversion to baseband (BB) is accomplished by mixing the receive signal with the fed back transmit signal,

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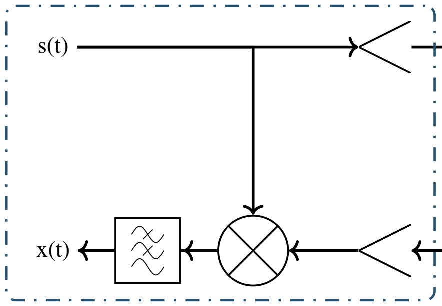
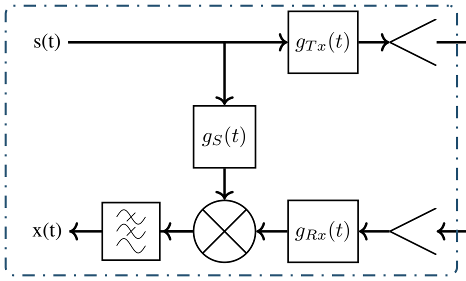


(a) Ideal stretch processor



(b) Non-ideal stretch processor

Fig. 1: Block diagram of stretch processor model

and subsequent integration (low-pass filtering). This receiver concept is denoted as stretch processor [2]. Figure 1(a) shows the corresponding block diagram. The resulting BB signal $x(t)$ is sampled by the analog-to-digital converter (ADC).

Apart from the ideal stretch processor shown in Fig. 1(a), an implemented one is non-ideal due to hardware impairments, causing signal distortions. For proper modelling of the radar system, such distortions have to be incorporated in the model. Hence, the ideal stretch processor model has to be extended. In the following, hardware caused signal distortions are assumed as linear time invariant (LTI) systems. Figure 1(b) shows the block diagram of the proposed model of a non-ideal stretch processor. The following LTI systems and signals are introduced

- Transmit signal $s(t) = U \cos[\phi(t)]$
- Baseband signal $x(t)$
- LTI system of the Tx channel $g_{Tx}(t)$
- LTI system of the Rx channel $g_{Rx}(t)$
- LTI system of the feedback channel $g_S(t)$
- LTI system of the transmission channel $h(t)$

Here, $g_{Tx}(t)$ models distortions of the transmit signal's phase and magnitude, whereas $g_{Rx}(t)$ describes distortions of the received signal. LTI system $g_S(t)$ incorporates distortions of the fed back transmit signal. In order to derive a radar system model it has to be clarified, how the introduced LTI systems are incorporated in the BB signal.

The BB signal $x(t)$ according Fig. 1(b) is given by

$$x(t) = \frac{1}{T} \cdot \int_t^{t+T} y_1(t') \cdot y_2(t') dt' \quad (5)$$

whereas $y_1(t)$ and $y_2(t)$ are convolution results

$$y_1(t) = s(t) * g_{Rx}(t) * h(t) * g_{Rx}(t) \quad (6)$$
$$y_2(t) = s(t) * g_S(t) \quad . \quad (7)$$

Convolution of the periodic LFM signal with an arbitrary LTI system with impulse response $g(t)$ is

$$s(t) * g(t) = U \cdot \int_{T_C} g(\tau) \cdot \cos[\phi(t-\tau)]\, d\tau \quad . \quad (8)$$

For a large time-bandwidth product and under narrowband conditions, hence $T_C \cdot B_C >> 1$ and $f_c >> B_C$ holds, the convolution can be approximated by

$$s(t) * g(t) \approxeq U \cdot U_G(f_t) \cdot \cos[\phi(t) + \phi_G(f_t)] \quad . \quad (9)$$

$U_G(f_t)$ and $\phi_G(f_t)$ denote magnitude and phase of the frequency response of $g(t)$, and $f_t = f(t)$ is the instantaneous frequency according to equation (3). Consequently, the convolutions in equation (6) and (7) are approximately

$$y_1(t) \approxeq U \cdot U_H(f_t) \cdot U_{G_{Rx}}(f_t) \cdot U_{G_{Tx}}(f_t) \cdot \cos[\phi(t) + \phi_{G_{Tx}}(f_t) + \phi_H(f_t) + \phi_{G_{Rx}}(f_t)] \quad (10)$$
$$y_2(t) \approxeq U \cdot U_{G_S}(f_t) \cdot \cos[\phi(t) + \phi_{G_S}(f_t)] \quad . \quad (11)$$

Plugging into the stretch processor model from equation (5) yields

$$\begin{aligned} x(t) &\approxeq \int_t^{t+T} A(t') \cdot \cos[\alpha_1(t')] \cdot \cos[\alpha_2(t')]\, dt' \\ &= \int_t^{t+T} \frac{A(t')}{2} \cdot \cos[\alpha_1(t') + \alpha_2(t')]\, dt' \\ &\quad + \int_t^{t+T} \frac{A(t')}{2} \cdot \cos[\alpha_1(t') - \alpha_2(t')]\, dt' \end{aligned} \quad (12)$$

whereas

$$A(t) = \frac{U^2}{T} \cdot U_{G_{Tx}}(f_t) \cdot U_H(f_t) \cdot U_{G_{Rx}}(f_t) \cdot U_{G_S}(f_t) \quad (13)$$
$$\alpha_1(t) = \phi_{G_{Tx}}(f_t) + \phi_{G_{Rx}}(f_t) + \phi_H(f_t) + \phi(t) \quad (14)$$
$$\alpha_2(t) = \phi(t) + \phi_{G_S}(f_t) \quad (15)$$

Because only the BB signal part is of interest, the integration in equation (12) is chosen such that the first term vanishes and the second term remains.

$$\begin{aligned} x(t) &\approxeq \frac{U^2}{2T} \cdot U_{G_{Tx}}(f_t) \cdot U_H(f_t) \cdot U_{G_{Rx}}(f_t) \cdot U_{g_S}(f_t) \\ &\quad \cdot \cos[\phi_{G_{Tx}}(f_t) + \phi_{G_{Rx}}(f_t) + \phi_H(f_t) - \phi_{G_S}(f_t)] \\ &= U_H(f_t) \cdot U_{Sys}(f_t) \cdot \cos[\phi_H(f_t) + \phi_{Sys}(f_t)] \end{aligned} \quad (16)$$

Using the Hilbert transform $\mathcal{H}\{.\}$, the real-valued BB signal $x(t)$ is transformed to an analytic, complex-valued signal $\tilde{x}(t)$.

$$\tilde{x}(t) = x(t) + \jmath\mathcal{H}\{x(t)\} = G(f_t) \cdot H(f_t) \quad (17)$$

Now, the hardware impairments of the radar system $G(f_t) = U_{Sys}(f_t) \exp[\phi_{Sys}(f_t)]$ and the transmission channel $H(f_t) = U_H(f_t) \exp[\phi_H(f_t)]$ are multiplicatively interrelated in the complex BB signal and hence are separable. In the following, $G(f_t)$ will be denoted as the radar system model, which is derived by calibration measurements.

III. Parameter Estimation

A. Data Model

As stated in the introduction, a model of the measured data is required for high-resolution parameter estimation. A radar system model was derived in the previous section, whereas no specific model on the transmission channel $h(t)$ was assumed so far. Here, the transmission channel is assumed to consist of paths, arising from propagation from Tx to Rx due to e.g. reflections from a target or the surrounding environment. Each path is described in terms of a delay τ_p and a weight factor γ_p. Assuming the superposition of P propagation paths, the transmission or propagation channel model is [3]

$$h(t) = \sum_{p=1}^{P} \gamma_p \cdot \delta(t - \tau_p) \quad . \tag{18}$$

According to equation (17), the transmission channel is included in the received signal by his frequency response at frequency f_t.

$$H(f_t) = \sum_{p=1}^{P} \gamma_p \cdot \exp\left[\jmath\varphi(f_t, \tau_p)\right] \tag{19}$$

A model for the complex-valued BB signal $\tilde{x}(t)$ is now given by plugging in the channel model from equation (19) in equation (17).

$$\tilde{x}(t) = G(f_t) \sum_{p=1}^{P} \gamma_p \exp\left[\jmath\varphi(f_t, \tau_p)\right] = \sum_{p=1}^{P} \gamma_p m(f_t, \tau_p) \tag{20}$$

The measurements are superimposed by noise $n(t)$. Here, additive white and complex Gaussian noise with power σ^2 and zero-mean is assumed. Hence, the model for the measurement data $y(t)$ becomes

$$y(t) = \tilde{x}(t) + n(t) = \sum_{p=1}^{P} \gamma_p \cdot m(f_t, \tau_p) + n(t) \quad . \tag{21}$$

Assuming the sampling of L data points by the ADC, the data model (21) can be represented in vector notation

$$\mathbf{y} = \mathbf{M}(\boldsymbol{\tau}) \cdot \boldsymbol{\gamma} + \mathbf{n} \quad \in \mathbb{C}^{L \times 1} \tag{22}$$

with $\boldsymbol{\tau} = \left[\tau_1 \dots \tau_P\right]^T$ and $\boldsymbol{\gamma} = \left[\gamma_1 \dots \gamma_P\right]^T$ comprising the delay and path weight for all P paths, respectively. Matrix $\mathbf{M}(\boldsymbol{\tau}) = \left[\mathbf{m}(\tau_1) \dots \mathbf{m}(\tau_P)\right]$ contains the sampled realisation of function $m(f_t, \tau_p)$ from equation (20).

B. Maximum-Likelihood Estimator

Considering the assumption of complex Gaussian noise, the distribution of the measurement vector $\mathbf{y}$ is

$$\mathbf{y} \sim \mathcal{CN}\left(\mathbf{M}(\boldsymbol{\tau}) \cdot \boldsymbol{\gamma}, \sigma^2 \cdot \mathbf{I}\right) \quad . \tag{23}$$

Following the ML method [4], a non-linear least-squares objective function can be derived, which has to be minimised for parameter estimation.

$$\hat{\boldsymbol{\tau}}, \hat{\boldsymbol{\gamma}} = \underset{\boldsymbol{\tau}, \boldsymbol{\gamma}}{\operatorname{argmin}} \ \|\mathbf{y} - \mathbf{M}(\boldsymbol{\tau}) \cdot \boldsymbol{\gamma}\|_2^2 \tag{24}$$

TABLE I: Basic radar settings

f_c	157 GHz
B_C	20 GHz
T_C	1024 µs
ADC sampling rate	4 MHz

Another estimation task is the selection of the model order, respectively the determination of the number of paths P. Several methods to determine the model order are known from literature [5], which will not be covered here. In the following, the number of paths is assumed as known.

Assuming that an estimated path results from a target reflection, the target's range is related to the path delay by $r = c \cdot \frac{\tau}{2}$. Furthermore, the target's radar cross section (RCS) ς is related to the path weight γ by the radar transmission equation [6], because the path weight's magnitude denotes the transmission attenuation.

$$|\gamma| = \frac{\lambda_c^2}{(4\pi)^3 \cdot r^4} \cdot \varsigma = \frac{1}{4\pi^3 \cdot f_c^2 \cdot c^2 \cdot \tau^4} \cdot \varsigma \tag{25}$$

Additionally, non-parametric processing is considered for parameter inference, in order to validate the ML estimator. Here, FFT of the measured data and subsequent peak search is employed, in order to gain the path's delay and magnitude.

IV. Measurements and Results

Measurements and parameter estimation were performed in order to verify the proposed radar system model for an existing radar device. The radar device described in [7] was utilised therefore. Basic radar settings are summarised in TABLE I. The measurements were performed in a laboratory environment and two different corner reflectors were sequentially deployed as targets. The corners have a RCS of $0.45\,\mathrm{dBsm}$ and $-10\,\mathrm{dBsm}$, and were placed $0.8\,\mathrm{m}$ and $1.6\,\mathrm{m}$ apart from the radar system. In total, 4 measurements were taken.

For the radar system under consideration, a real-valued BB signal is present. Hence, transformation to a complex-valued analytic signal is conducted first. Also, strong DC-components are reduced by high-pass filtering. The resulting range spectra, denoted as pre-processed in the following, are depicted in Fig. 2. The main peaks coincide with the propagation path from target back scattering. Additionally, 3 other paths are visible, which cannot be related to reflections from the target or the surrounding environment. Hence, they are denoted as ghost paths. These ghost paths occur due to hardware related signal distortions [7]. However, the radar model from Section II does not cover these distortions, because the ghost paths' occurrence depends on the target location, as visible by comparing Fig. 2(a) and Fig. 2(b). Hence, in order to use the presented data model for parameter estimation, another signal processing step is necessary.

Limitation to the frequency band from $156.8\,\mathrm{GHz}$ to $159.6\,\mathrm{GHz}$ was conducted to remove distorted signal parts, causing the ghost paths. The processing results are denoted as post-processed and are shown in Fig. 2. As visible, the ghost

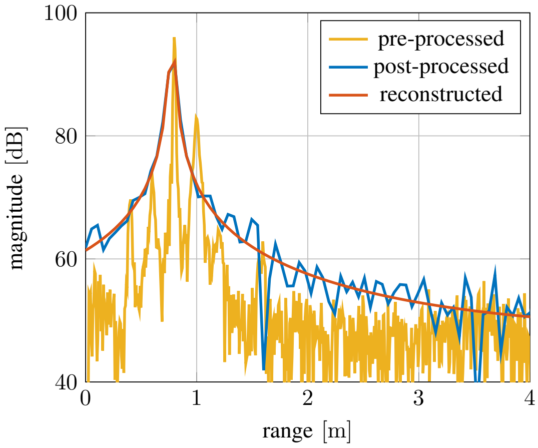
TABLE II: Inferred parameter values for various measurement settings

range			RCS		
setting	estimate (ML)	estimate (FFT)	setting	estimate (ML)	estimate (FFT)
0.8 m	0.81 m	0.81 m	0.45 dBsm	0.60 dBsm	3.15 dBsm
0.8 m	0.79 m	0.80 m	−10 dBsm	−9.93 dBsm	−9.97 dBsm
1.6 m	1.65 m	1.66 m	0.45 dBsm	0.41 dBsm	1.80 dBsm
1.6 m	1.64 m	1.66 m	−10 dBsm	−9.85 dBsm	−9.77 dBsm

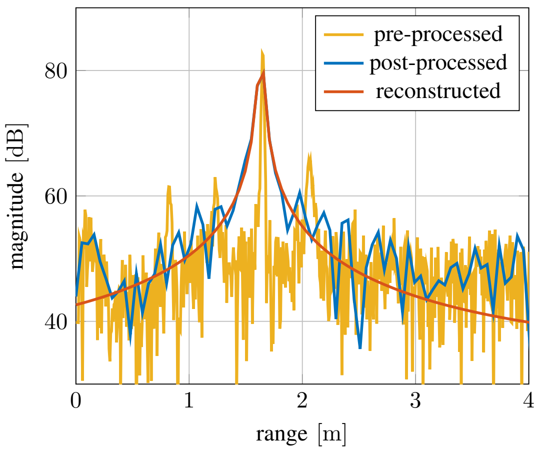
paths are suppressed. Subsequently, the ML and FFT based estimators as described in Section III were applied.

Inferred parameter values, assuming model order $P = 1$, are depicted in TABLE II. The ML estimates fit well with the scenario settings and the FFT based results. On the other hand, RCS estimates from FFT based processing are slightly biased, because the peak's magnitude in the range spectrum is reduced due to the bandwidth limitation. Using equation (20), the BB signal was reconstructed employing the estimated parameters. The results are depicted in Fig. 2 and are denoted as reconstructed. Obviously, the reconstructed range spectra are in good accordance to the post-processed ones.

Summarised, the proposed radar system model is a proper description of the utilised radar device after the band limitation.



(a) Range 0.8 m and RCS −10 dBsm



(b) Range 1.6 m and RCS −10 dBsm

Fig. 2: Range spectra according to various measurement settings and for different processing steps

V. Conclusion

High-resolution parameter estimation for a chirp sequence radar was considered to enhance the radar's resolution capability. In order to find a proper data model for the estimation task, the radar system architecture, which is basically a stretch processor, was analysed. The derived radar system model incorporates hardware impairments, which are assumed as LTI systems. Calibration measurements were conducted to parametrise this radar system model.

A high-resolution estimator was derived from the ML principle, estimating delay and amplitude of propagation paths, which are possible target back scattering. This estimator was validated using measurements with a real radar device. It turned out, that ghost paths are present in the measurements, which occurrence is not covered by the proposed radar system model. Hence, bandwidth reduction was conducted to exclude the ghost paths from the measurements. The ML estimation results fit well with the FFT based results and the measurement scenario's setup. Hence, the proposed radar system model properly describes the used radar device after band limitation.

Disadvantageously, band limitation reduces the resolution capability. Overcoming this issue necessitates an improved radar system model. Hence, further research will be spent towards the investigation of the ghost paths and their incorporation in an improved radar system model. Advantageously, complicated system models can be easily incorporated in the parametric signal processing and estimation framework.

References

[1] A. Tarantola, *Inverse Problem Theory and Methods for Model Parameter Estimation*. Society for Industrial and Applied Mathematics (SIAM), 2005.

[2] M. A. Richards, *Fundamentals of Radar Signal Processing*, 2nd ed. Madison: McGraw Hill Professional, 2013.

[3] J. Li and P. Stoica, *MIMO Radar Signal Processing*. New York: John Wiley & Sons, 2008.

[4] H. L. Van Trees, *Optimum Array Processing - Detection, Estimation, and Modulation Theory*. New York: John Wiley & Sons, 2004.

[5] P. Stoica and Y. Selen, "Model-order selection: a review of information criterion rules," *IEEE Signal Processing Magazine*, vol. 21, no. 4, pp. 36–47, July 2004.

[6] M. I. Skolnik, *Radar Handbook*, 2nd ed. New York: McGraw Hill, 1990.

[7] M. Hitzler, S. Saulig, L. Boehm, W. Mayer, W. Winkler, N. Uddin, and C. Waldschmidt, "Ultracompact 160-GHz FMCW Radar MMIC With Fully Integrated Offset Synthesizer," *IEEE Transactions on Microwave Theory and Techniques*, vol. 65, no. 5, pp. 1682–1691, May 2017.

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