Comparison of 2D and 3D Compressed Sensing for High-Resolution TDM-MIMO Radars

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Abstract— **In order to enhance the resolution of radar sensors, the sampling of the different measurement domains range, Doppler, and direction-of-arrival can be improved. This, for example, leads to the application of a sparse array for an increased angular resolution and an enhanced multiple-input multiple-output switching scheme for an increased maximum Doppler velocity. In this paper, an investigation is performed in order to determine the dimension where undersampling is most effective. As the measurement dimensions are linked, two different reconstruction approaches of the subsampled data are presented and evaluated. Measurements validate the effective reconstruction of the subsampled data.**

Keywords— **MIMO radar, direction-of-arrival estimation, sparse arrays, compressed sensing, automotive radar**

I. Introduction

For future driver assistance systems a radar sensor with high-resolution capabilities in the three measurement domains range, Doppler, and direction-of-arrival (DoA) is required [1]. This leads to a large bandwidth for better range resolution, a long measurement duration for an enhanced Doppler resolution, and to more antenna elements increasing the whole aperture for an improved angular resolution. Ideally, all three domains are fully sampled leading to a huge hardware effort.

This motivates to optimise the sampling in the different domains, e.g., to increase the antenna array by replacing a uniform linear array (ULA) with a minimum-redundancy array [2] or sparse arrays in general. The resulting high sidelobes are successfully mitigated in publications by a compressed sensing-based evaluation of the arrays [3], [4].

The hardware effort can be further reduced by the application of a multiple-input multiple-output (MIMO) approach requiring orthogonal waveforms [5]. The simplest choice [6] is the time-division multiplexing (TDM). The drawback of the reduced maximum unambiguous Doppler velocity can be compensated by randomly activating the transmitters and restoring the missing data afterwards [7].

Additionally, several receiving channels can be sampled by the same analog-to-digital converter (ADC) leading to a switched antenna array [8]. To prevent phase errors due to the serial sampling, the switching can be realised in a random order with a reconstruction of the omitted samples [9], [10].

In order to optimise the overall resolution parameters, a combination of the approaches in the different measurement domains seems reasonable. A randomly undersampling in all domains and a reconstruction by compressed sensing already showed promising results in numerical simulations [11]. In this case the different domains are linked to each other, e.g., for a specific antenna array, the required number of transmitters leads to an undersampling in the Doppler domain. Instead of evaluating each measurement domain separately, combined approaches seem promising as later shown.

In this paper a two- and three-dimensional undersampling of the measurement domains is applied. In a simulation, two different reconstruction schemes are compared with respect to their performance. As the angular separation is a challenging demand [1], two targets in the same range-Doppler cell only with different azimuth angles are selected. Hence, for verification two different DoA measurements are shown, proofing the effectiveness of the proposed approaches.

II. Undersampling: Different Sparseness Domains

For the compressed sensing-based reconstruction of a signal sampled below the Nyquist rate, certain conditions must be fulfilled [12]. An important requirement is that the signal is sampled randomly and not with a periodic pattern. Additionally, the signal should be sparse in the image domain. For a radar signal this means that the spectrum should only contain a limited number of significant frequency components. This is true as the used chirp-sequence modulation [13] separates targets in different range-Doppler cells.

A reduction in a certain measurement domain, e.g., missing antenna elements, leads to missing samples. In the following this reduction is called *sparseness* of a signal, which should not be mistaken with the sparsity of the signal.

A. Angular Domain: Sparse Arrays

A MIMO radar with four transmitters and eight receivers is considered. An optimised sparse array is usually designed with a genetic algorithm considering the tradeoff between a large aperture and the ambiguity-free region. Thus, the design is determined for the given boundary conditions as described in [4]. Fig. 1 shows that the sparse array nearly doubled the virtual aperture size compared to the corresponding ULA. This increases the angular resolution from 4.5° to 2.4°. As the element positions are always multiples of $\frac{\lambda}{2}$, the gaps in the aperture can be considered as missing elements defining the sparseness in the angular domain.

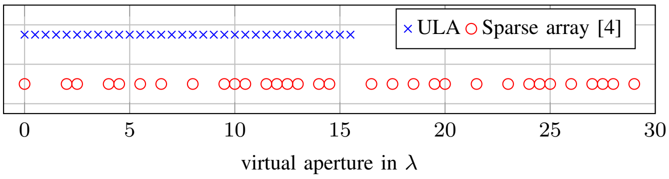


Fig. 1. Comparison of the virtual aperture of a ULA (×) and a sparse array (○) with the same number of elements. This optimised array was designed in [4].

B. Velocity Domain: Enhanced Doppler Unambiguity

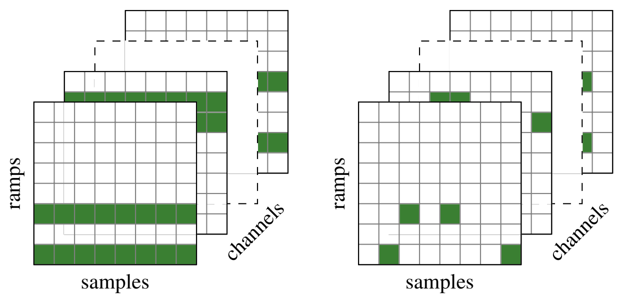
The applied TDM-MIMO scheme requires that the transmitters are consecutively active. Usually, the assignment of transmitted frequency ramps to the respective transmitters reduces the maximum unambiguously detectable Doppler frequency as the frequency ramp repetition interval increases [14]. A random assignment of frequency ramps to the transmitters allows to restore the not transmitted ramps. After reconstruction, this can be understood as if all transmitters are active at the same time not reducing the maximum Doppler frequency. The radar has four transmitters, which leads to an undersampling by a factor of four.

C. Range Domain: Required ADC Number Reduction

The modulation parameters together with the link budget define the maximum achievable range leading to the required sampling rate of 20 MHz. Although each single receiver has its own ADC, the maximum sampling frequency of the system is 100 MHz. Hence, at maximum four receivers could share one ADC together with a fast switch. Therefore, an undersampling of the range domain by a factor of four is considered for the simulation.

D. Undersampling Combinations

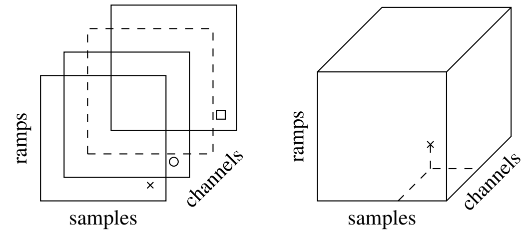
The sparseness of the three dimensions can be combined, however, only two combinations are considered useful. As the DoA estimation is crucial for the separation of close targets in the same range-Doppler cell, the sparse array is applied. The radar sensor operates in MIMO mode, hence, the TDM scheme with the randomly active transmitters is used. This is equal to a data rate reduction to a fourth. The combination of both schemes is referenced as *S2D* and depicted in Fig. 2 (a).



(a) 2D sparseness: velocity and DoA (*S2D*)

(b) 3D sparseness: range, velocity, and DoA (*S3D*)

Fig. 2. The two considered combinations of sparseness for the simulation. The present samples are marked in green (■).



(a) 2D reconstruction of each channel, afterwards 1D for each target cell, named *R2D*.

(b) 3D reconstruction of the whole data cube at once. Referenced as *R3D*.

Fig. 3. The two different reconstruction approaches. In (a) it is a two-step process, in (b) the data cube is restored directly.

The second option additionally reduces the number of ADCs resulting in a sparseness in all three measurement dimensions. It is depicted in Fig. 2 (b) and referenced as *S3D*.

III. Reconstruction Approaches

Missing samples lead to strong artefacts in spectrum, which must be considered by the reconstruction algorithm. They originate from the convolution of the targets with the missing sample pattern. To regain a fully sampled signal the iterative method with adaptive threshold (IMAT) [15] is applied.

The missing signal samples are assumed to be zero and then the signal is transformed to the spectrum. As the sparsity assumption is valid, the maximum frequency component is the first threshold. Every frequency component above the threshold is used for the transformation back in the time domain yielding a first assumption of the missing samples. In consecutive iterations the threshold is lowered and the artefacts are reduced. As a stopping criterion a safety margin of several dB is added to the noise floor [10] to avoid that the threshold is lowered into the artefacts.

An investigation of the minimum required number of iterations for a sufficient reconstruction is performed in [16]. In the following two different reconstruction schemes are presented.

A. Two-Step Reconstruction by Channel (R2D)

Due to the TDM scheme with randomly active transmitters and a possible ADC reduction, samples are missing in the signal of each receive channel, cf. Fig. 2. A straightforward approach is to first reconstruct the missing samples of each available channel. In this case the transformation to the spectrum is a two-dimensional Fourier transform. Afterwards, targets are detected and range-Doppler cells of interest are determined. For each selected cell a one-dimensional reconstruction is applied, as the sparse array leads once again to an undersampling of the angular domain. This procedure is depicted in Fig. 3 (a) and referenced as *R2D*.

B. One-Step Reconstruction of Data Cube (R3D)

A simple DoA estimation can be performed with a Fourier transform if the antenna elements are placed on an equidistant grid. Otherwise, the Fourier transform has to be adapted accordingly. This motivates the approach to directly calculate

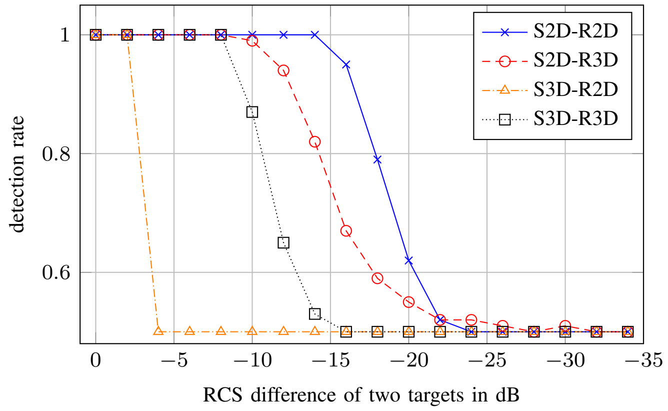


Fig. 4. Simulation of the detection rate for two targets in the same range-Doppler cell with different RCS levels. A sparse array with a random TDM scheme (*S2D*) and with an additional ADC reduction (*S3D*) is considered. The reconstruction is performed in a two-step process first for each channel and then for the sparse array (*R2D*) or in one-step for the whole data cube (*R3D*). With a detection rate of 0.5 only one target is detected.

a three-dimensional Fourier transform on the whole data cube as shown in Fig. 3 (b). In this case the measured data of all channels are directly evaluated which can lead to a processing gain and, hence, a better detection of weak targets. This one-step approach is referenced as *R3D*.

IV. Simulative Comparison

The chirp-sequence radar parameters used for the simulation are listed in Table 1. The application of a sparse array increases the sidelobes and complicates the detection of targets with substantially different radar cross-sections (RCS) in the same range-Doppler cell. This effect significantly influences the detection capabilities. Considering automotive scenarios, a pedestrian in the close vicinity of a vehicle can lead to such a large difference. Therefore, the simulation investigates the maximum RCS difference of two targets in the same cell for a successful detection using different undersampling combinations and reconstruction approaches. A DoA difference of 20° is assumed as in this case the second target may vanish in the sidelobes.

At first, the sparse array together with the random TDM scheme is investigated (*S2D*). The two-step reconstruction (*R2D*) leads to a perfect reconstruction until the RCS difference grows larger than 14 dB () as shown in Fig. 4. This limit mainly depends on the sidelobe level of the sparse array.

The same undersampling combination is reconstructed with the one-step approach (*R3D*) (), which is only ideal until

Table 1. Specifications of Chirp-Sequence Radar Parameters

Parameter	Value
carrier frequency f_c	76.5 GHz
bandwidth B	2 GHz
chirp duration T_c	50 µs
chirp repetition time T_r	60 µs
sampling frequency f_s	20 MHz
number of chirps L	256

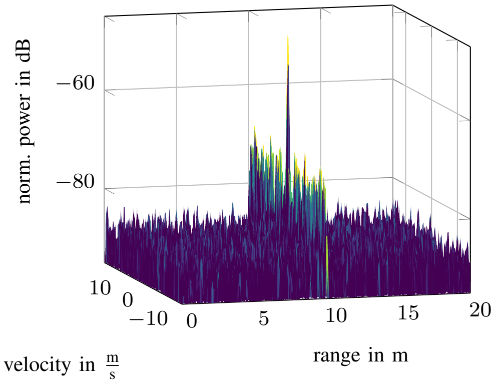


Fig. 5. Spectrum of a single receive channel without reconstruction shows a dominant artefact level arising from the missing frequency ramps [10].

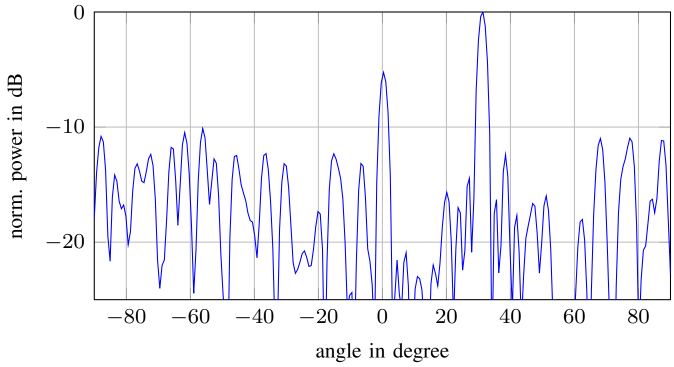


Fig. 6. The DoA spectrum without reconstruction of the sparse array shows high sidelobes for a RCS difference of 5 dB. A small safety margin is required.

a difference of roughly 10 dB. Although this approach should lead to a better performance due to the increased processing gain using all channels, the opposite effect occurs. The reason is that the missing frequency ramps lead to a dominant artefact level in the spectrum of a single channel as can be seen in Fig. 5. To prevent the threshold from entering the artefact level, a large safety margin of 10 dB is applied. Otherwise random frequencies would be intensified and false targets would be the outcome. Since the reconstruction of the sparse array is done inside the three-dimensional reconstruction, the large safety margin prohibits the detection of weak targets. The DoA spectrum of the sparse array without reconstruction is shown for a RCS difference of only 5 dB in Fig. 6. In the *R2D* reconstruction the safety margins can be chosen independently.

The additional ADC reduction (*S3D*) reduces the amount of available samples by a factor of four for each channel. This worsens the detection capabilities in case of the two-step detection () to a possible RCS difference of only 2 dB.

The three-dimensional reconstruction (), cf. Fig. 4, outperforms the two-step reconstruction and can reliably detect a RCS difference of 8 dB. The reason for the increased performance is, that the data of all 32 channels are used for reconstruction. Although only $\frac{1}{16}$ of the data is present, the artefact level is reduced compared to the *S2D* case due to the randomly missing samples. This behaviour is discussed in [10].

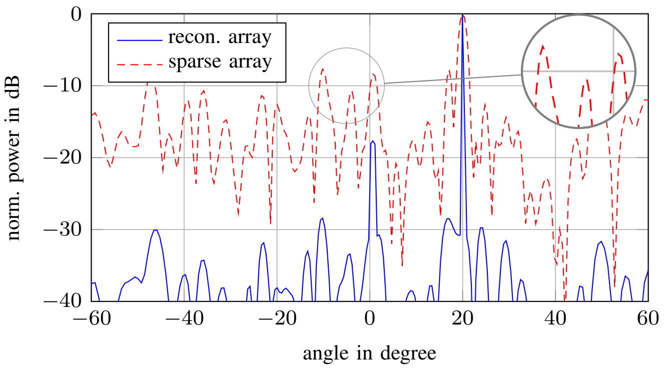


Fig. 7. Two targets with a RCS difference of 8.9 dB placed at 0° and 20°, respectively. DoA estimation before (- - - -) and after (——) reconstruction.

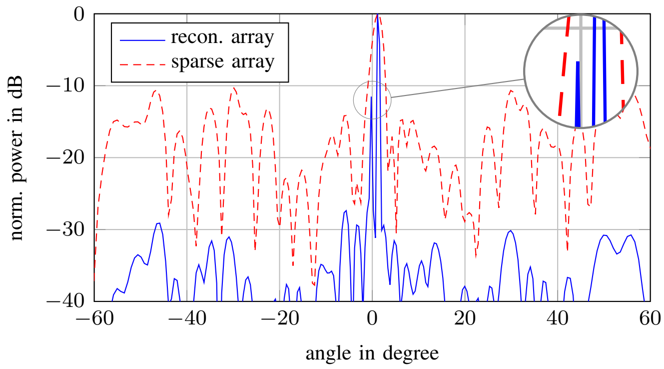


Fig. 8. Two targets separated by 2.4° with a RCS difference of 9.9 dB. DoA estimation before (- - - -) and after (——) reconstruction.

V. Measurement Results

In an automotive environment the dynamic range of RCS differences can be huge, e.g., a truck next to a pedestrian. This leads to the choice to only use the random TDM scheme together with the sparse array to achieve a maximum possible RCS difference. Although this approach (*S2D-R2D*) requires several iterations for every channel, the memory requirement is much lower compared to the three-dimensional reconstruction. In this case a Fourier transform is required for every range-Doppler cell which is usually calculated with a zero-padding leading to a huge amount of samples.

For verification purpose, two corner reflectors are placed in the same radial distance to the radar in an anechoic chamber. The maximum reliably detectable difference in RCS was roughly 10 dB, which is approximately 4 dB less than expected from the simulation. Nearly the same RCS difference is achieved if the classic TDM scheme without randomly active transmitters is applied, i.e. only an angular undersampling is present, cf. [4]. Hence, the crucial undersampled domain is the angular domain justifying the simulation considering only the DoA performance of close targets.

Two targets are placed at 0° and 20° in Fig. 7. Without reconstruction, the sidelobe in close vicinity to the target at 0° has an even higher power. Only after the successful reconstruction, the two targets can be distinguished from noise.

In Fig. 8 the two targets are placed with an angular difference of only 2.4° and a RCS difference of roughly 10 dB. Without reconstruction the second target is not visible at all, only after reconstruction it is detectable. The ideal angular resolution exists due the fact that a zero-padding to 256 values is applied before reconstruction. This can be considered as a virtual extension of the array increasing the aperture.

VI. Conclusion

To achieve high-resolution capabilities for an automotive radar sensor, the different measurement domains range, Doppler, and DoA can be undersampled. In this paper, the performed investigation showed that an undersampling in the angular domain, i.e. a sparse array, and a random TDM scheme leads to optimal results concerning the angular separation of close targets. Two different reconstruction schemes are compared showing that the two-step reconstruction of each channel matrix first and afterwards the undersampled DoA vector leads to better results compared to the other approach. Measurement results show that two targets with a RCS difference of nearly 10 dB in the same range-Doppler cell can be distinguished.

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